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Cooperative Data Protection in Topology-Aware Decentralized Storage Networks

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Outline

- **Introduction**
  - Motivation and Model
  - Existing work
- **Cooperative Data Protection**
  - ECC hierarchy
  - Single-level cooperation
- **Multi-level cooperation**
  - Cooperation graphs and compatible graphs
  - Construction over compatible graphs
- **Conclusion**

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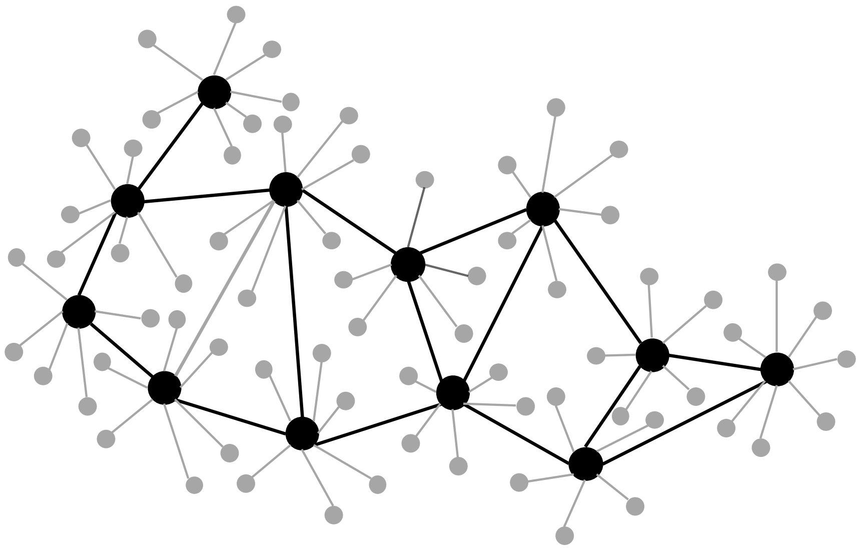
Motivation

- **Invention of Blockchain technology makes the concept of "decentralization" popular**
  - Higher privacy
  - Better scalability and flexibility
  - Economically attractive



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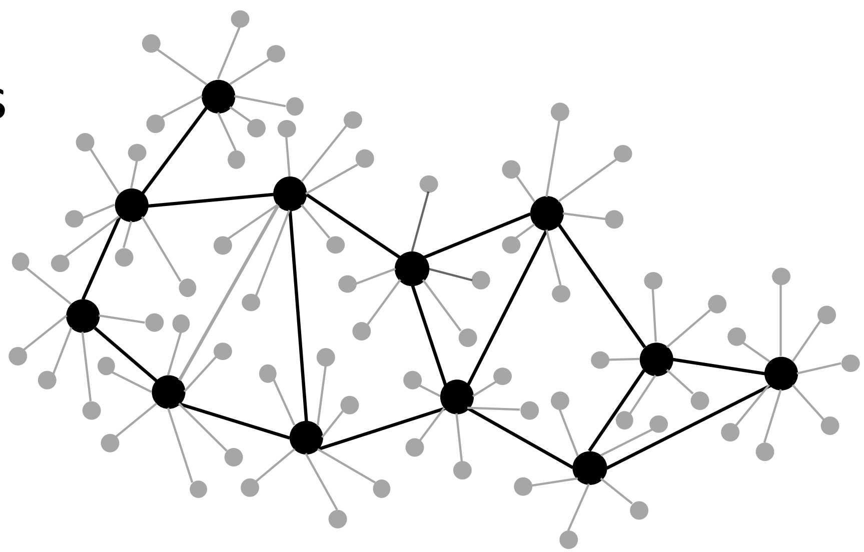
  - Decentralized storage networks
  - Masterless coded distributed computation
  - Federated learning
  - Wireless sensor networks



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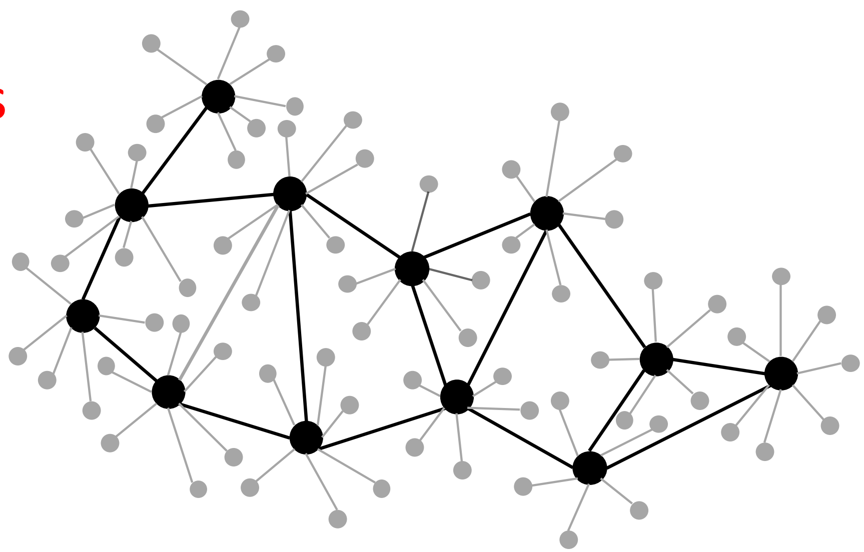
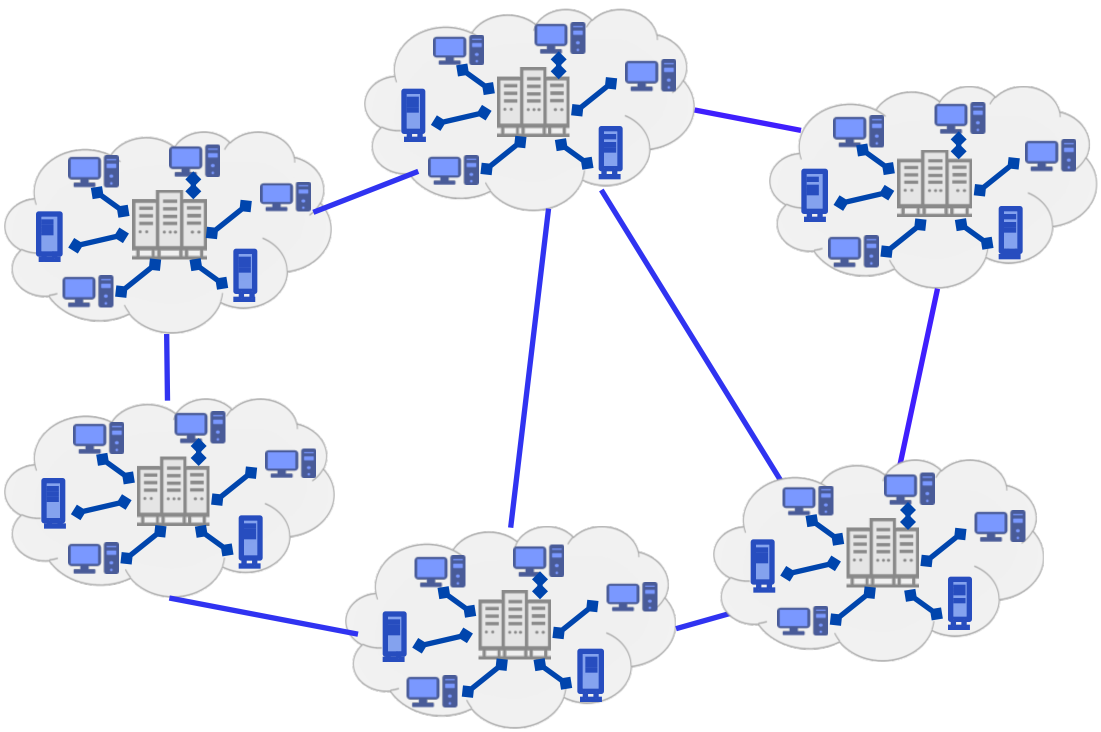


Dynamic DSNs: Structures

- **DSN: distributed local clouds**
  - Clusters of nodes
    - Local messages are encoded and stored distributively among local nodes



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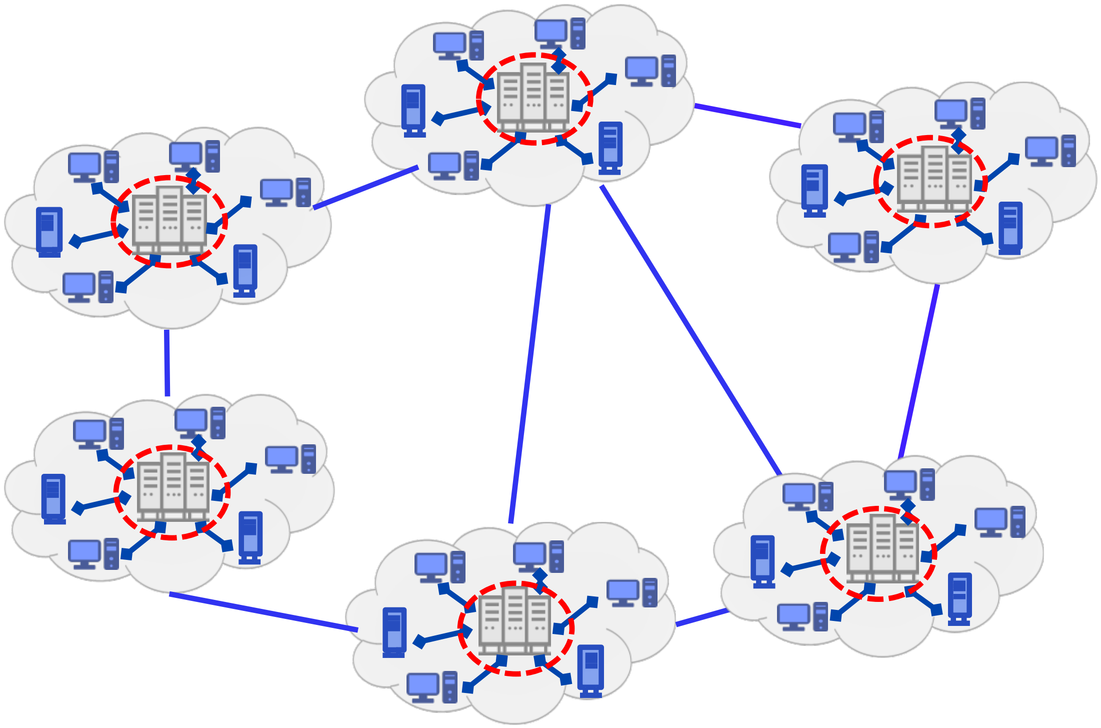

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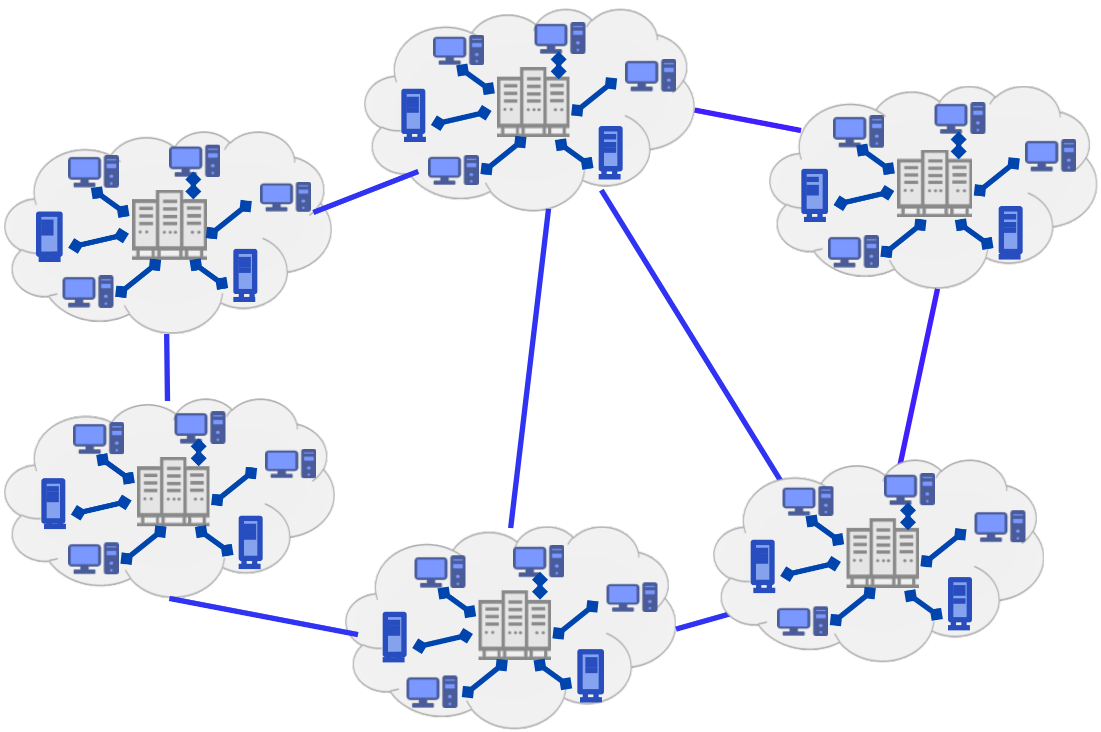
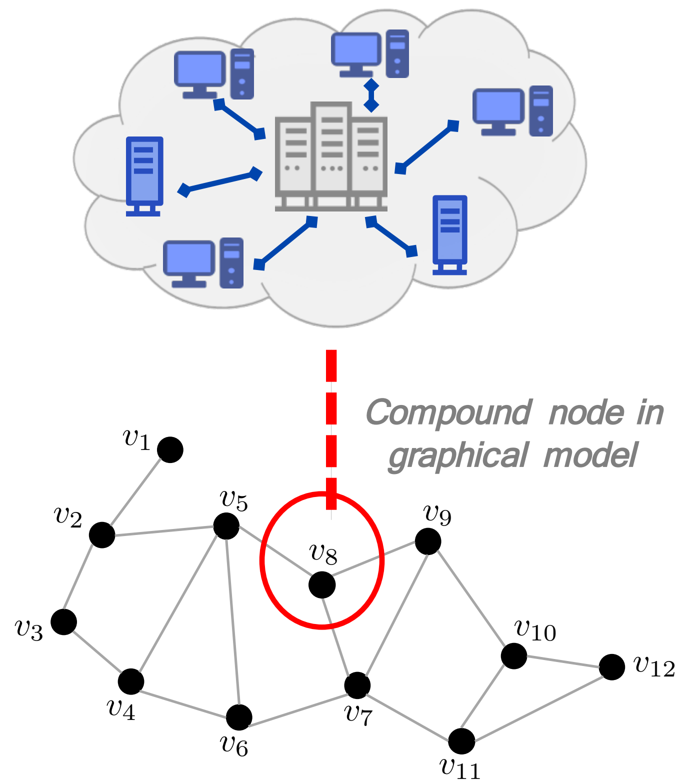
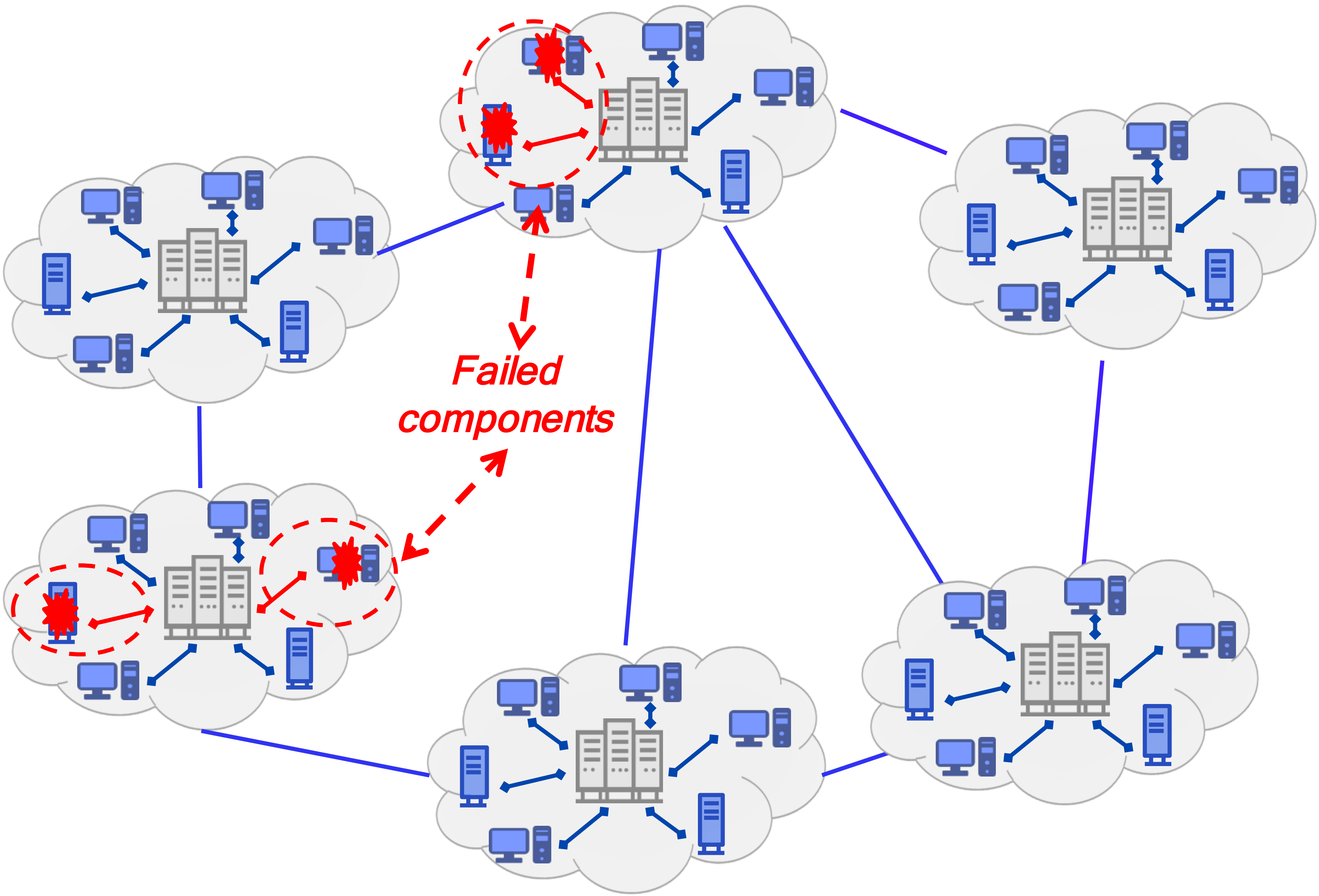




Dynamic DSNs: Practical Concerns



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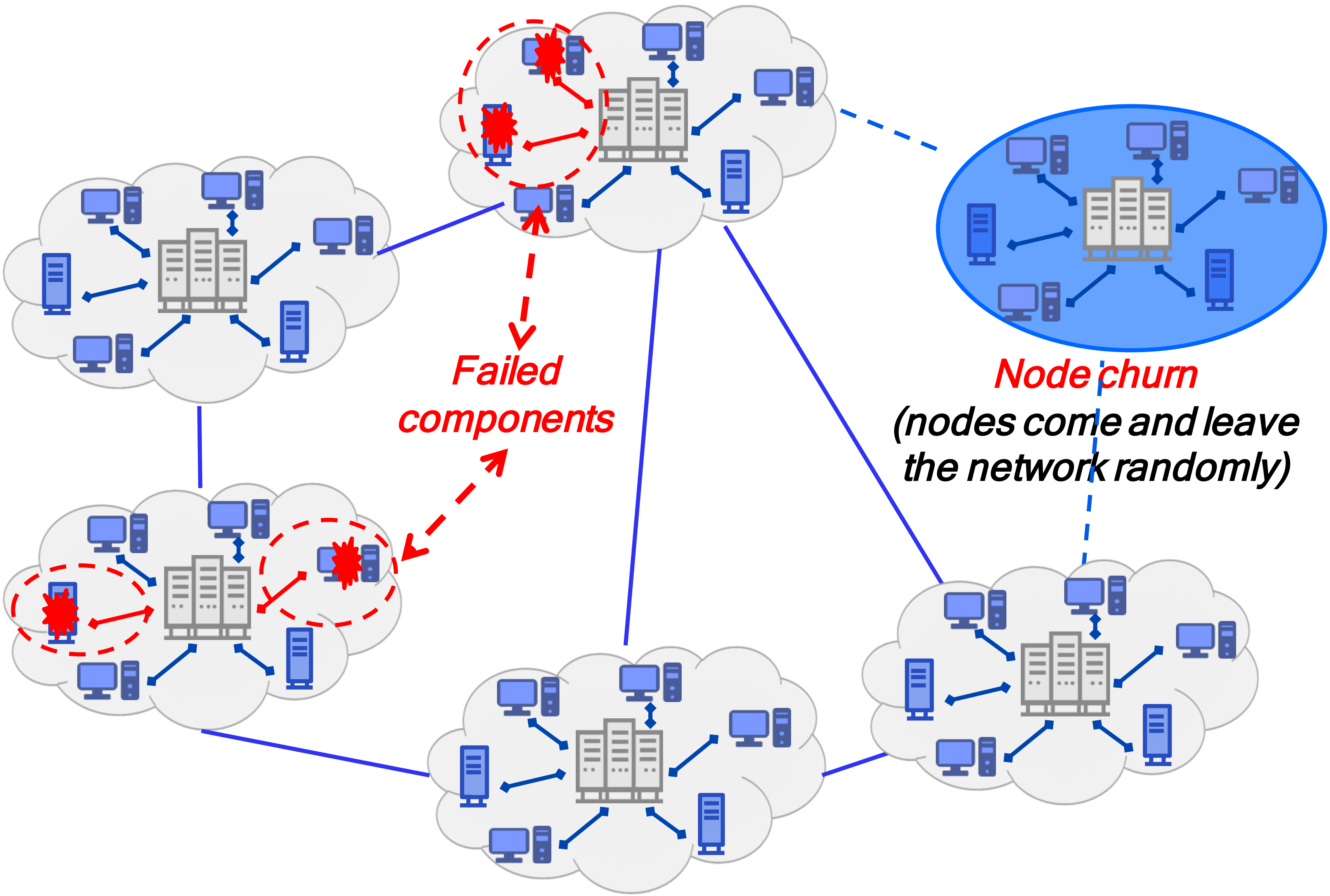
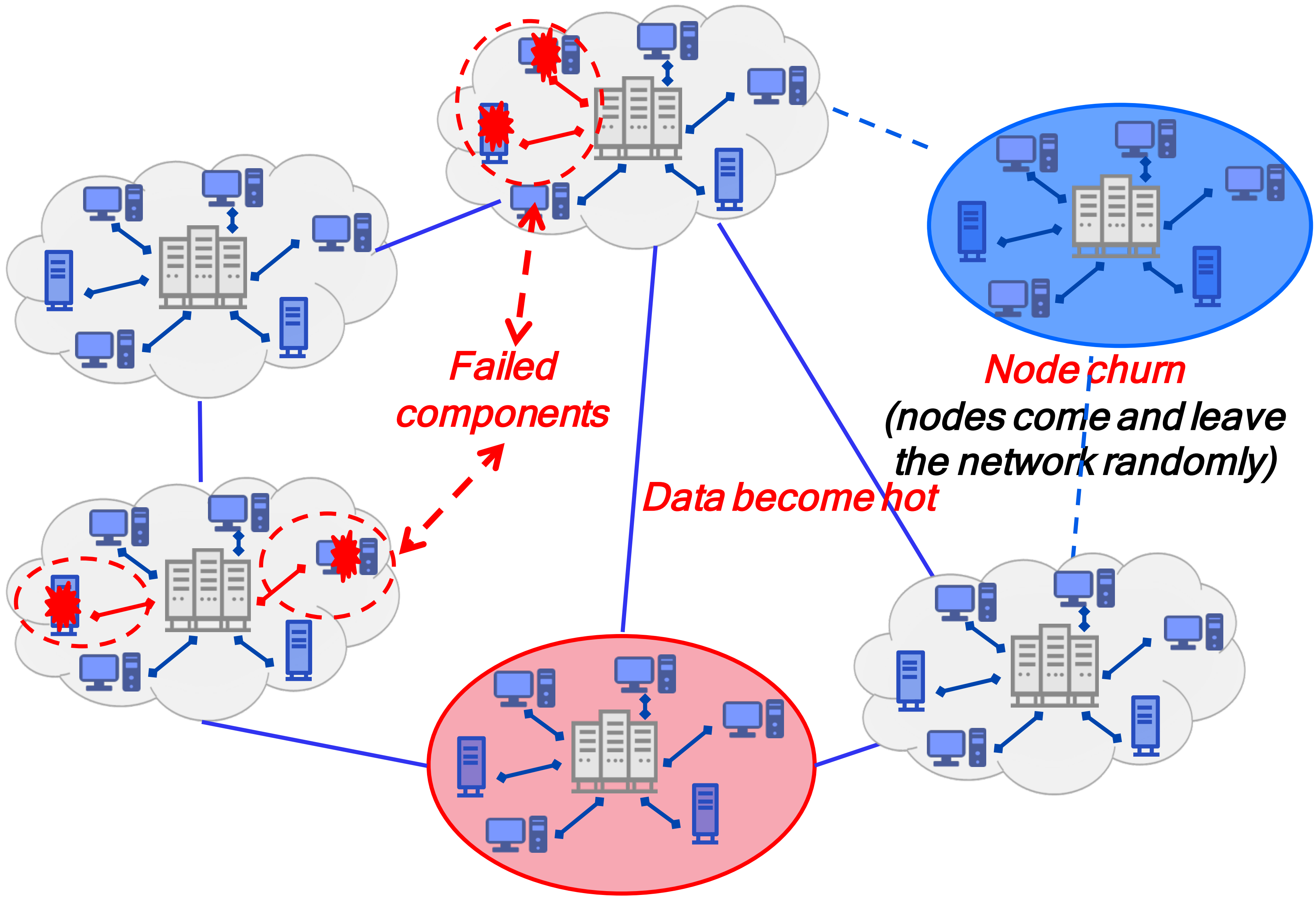


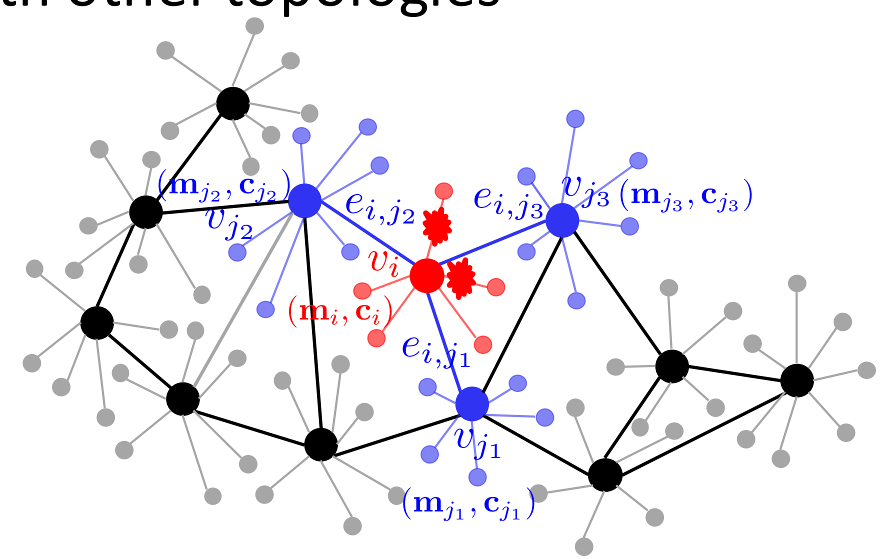
Dynamic DSNs: Practical Concerns



Desired Properties: Reliability and Latency

- **Hierarchical erasure correction**
  - Each node provides different levels of robustness for the codeword stored at it through accessing different sets of nodes
    - Trade-off between reliability and latency

- **Topology-awareness**
  - Schemes optimized for DSNs with a specific topology can result in bad performance in DSNs with other topologies
    - Intrinsic heterogeneity
    - Latency for inter-cloud communication can be much higher than that of intra-cloud communication



Desired Properties: Scalability and Flexibility

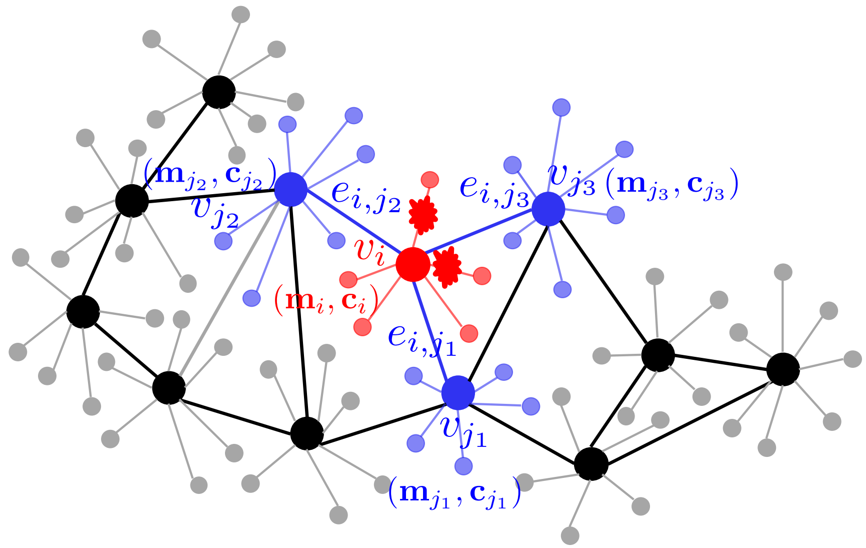
- **Scalability**
  - To support node churn in DSN
  - Expand the backbone network to accommodate additional workload without rebuilding the entire infrastructure

- **Flexibility**
  - To support the dynamic nature of the usage rate of data
  - Split a local cloud into smaller clouds if the data stored in it become hot

Our goal is to construct topology-aware coding schemes that provide hierarchical erasure correction at each node and simultaneously support scalability and flexibility

Abstract Model: Parameters

- **A DSN is modeled as a graph** $G(V,E)$
  - Number of master nodes: p
  - Message (codeword) stored at v_i: $\mathbf{m}_i(\mathbf{c}_i)$; lengths k_i (n_i)
  - Redundancy of $\mathbf{c}_i$: $r_i = n_i - k_i$
  - $\mathbf{m} = (\mathbf{m}_1, \mathbf{m}_2, \cdots, \mathbf{m}_p)$
  - $\mathbf{c} = (\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_p)$



Existing Literature

- **Distributed storage**[1]
  - No explicit consideration of clustering nature of network nodes

- **Multi-rack storage**[2-8]
  - Network topologies are predetermined
  - Capacities of the communication links are typically considered to be the same

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[1] A. G. Dimakis et al., "*Network coding for distributed storage systems*", IEEE Trans. Inf. Theory, vol. 56, no. 9, pp. 4539-4551, 2010

[2] Z. Kong et al., "*Decentralized coding algorithms for distributed storage in wireless sensor networks*", IEEE JSAC, vol. 28, no. 2, pp. 261-267, 2010

[3] M. Ye et al., "*Cooperative repair: Constructions of optimal MDS codes for all admissible parameters*", IEEE Trans. Inf. Theory, vol. 65, no. 3, pp. 1639-1656, 2018

[4] N. Prakash et al., "*The storage versus repair-bandwidth trade-off for clustered storage systems*", IEEE Trans. Inf. Theory, vol. 64, no. 8, pp. 5783-5805, 2018

[5] J. Li et al., "*Tree-structured data regeneration in distributed storage systems with regenerating codes*", IEEE INFOCOM, 2010

[6] Y. Wang et al., "*Non-homogeneous two-rack model for distributed storage systems*", IEEE INFOCOM, 2014

[7] H. Hou et al., "*Rack-aware regenerating codes for data centers*", *IEEE Trans. Inf. Theory, 2019*

[8] Z. Chen et al., "*Explicit constructions of MSR codes for clustered distributed storage: the rack-aware storage model*", [Online]. Available: https://arxiv.org/abs/1901.04419, 2019

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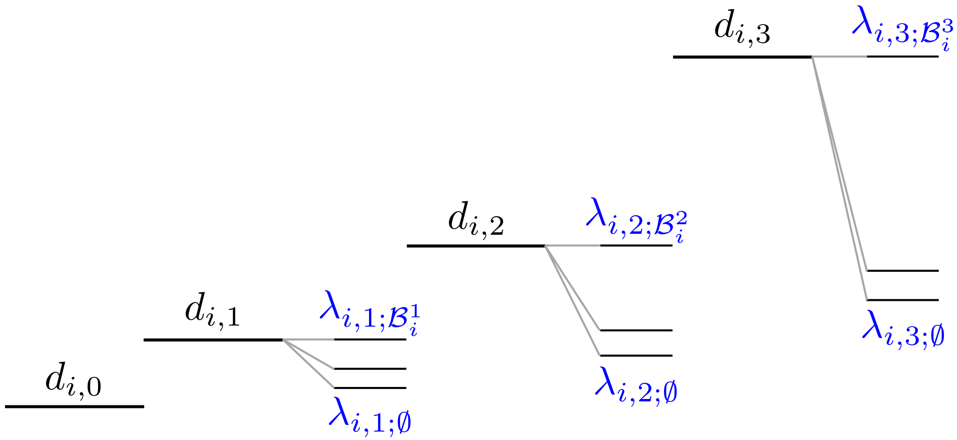
ECC Hierarchy

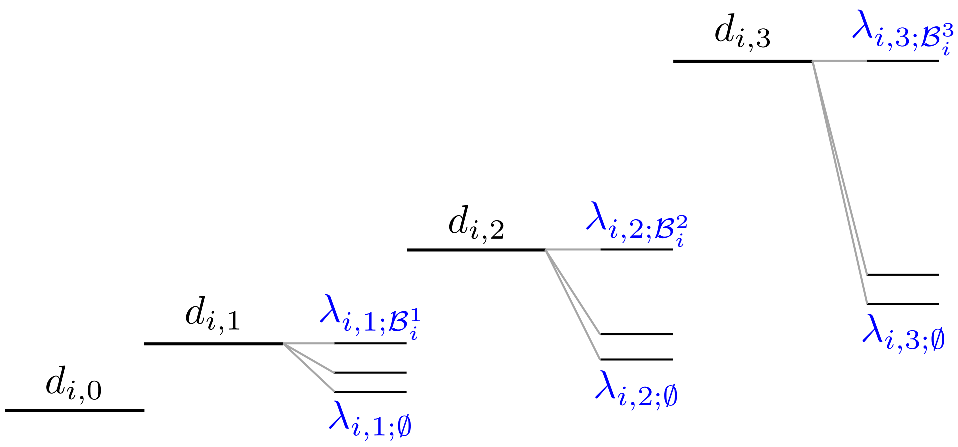
ECC hierarchy describes the erasure correction (EC) capabilities of nodes while cooperating with different sets of other nodes



ECC Hierarchy

ECC hierarchy describes the erasure correction (EC) capabilities of nodes while cooperating with different sets of other nodes

- **Cooperation at each node**
  - ECC hierarchy at v_i: $\mathbf{d}_i = \left(d_{i,0}, d_{i,1}, \cdots, d_{i,L_i}\right)$
    - $\emptyset \subset \mathcal{A}_i^1 \subset \mathcal{A}_i^2 \subset \cdots \subset \mathcal{A}_i^{L_i} \subseteq V, \{\mathcal{B}_i^l\}_{1\leq l \leq L_i}, \mathcal{A}_i^1 \cap \mathcal{B}_i^l = \emptyset$
    - $d_{i,l}$: EC capability at v_i if nodes in $\mathcal{A}_i^l \cup \mathcal{B}_i^l$ are recovered



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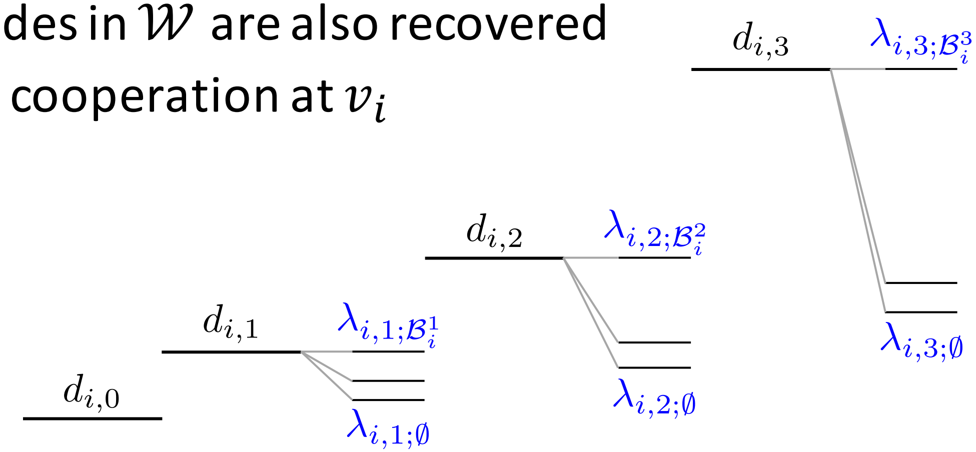
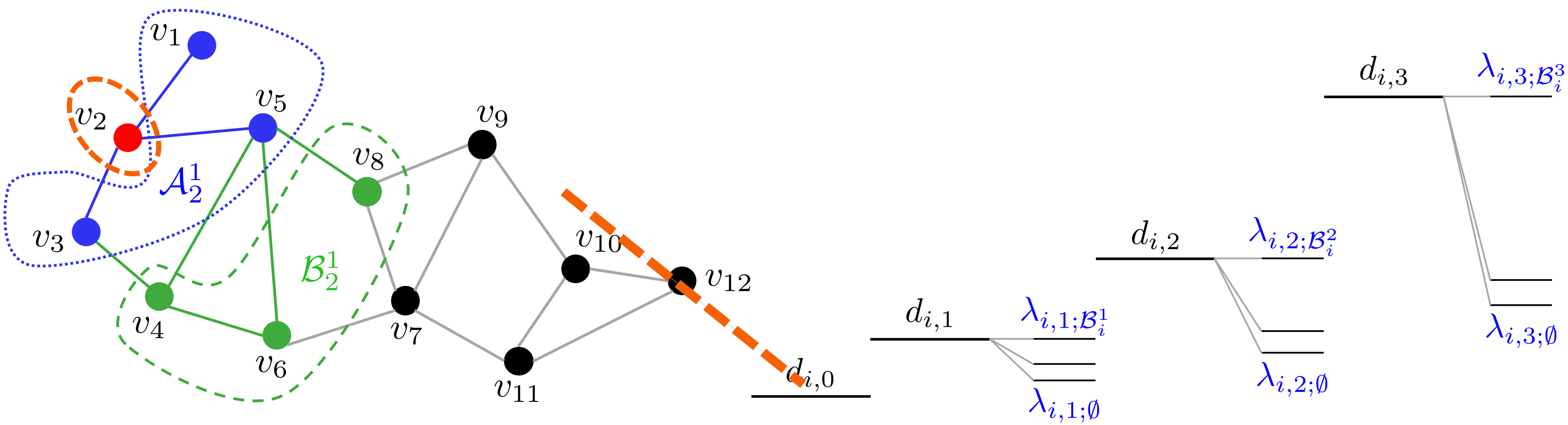
    - $d_{i,l}$: EC capability at v_i if nodes in $\mathcal{A}_i^l$ are recovered
  - Finer EC capability: $d_{i,l} = \left(\lambda_{i,l;\mathcal{W}}\right)_{\emptyset \subseteq \mathcal{W} \subseteq \mathcal{B}_i^l}$
    - $\lambda_{i,l;\mathcal{W}}$: EC capability while nodes in $\mathcal{W}$ are also recovered and involved in the l-th level cooperation at v_i



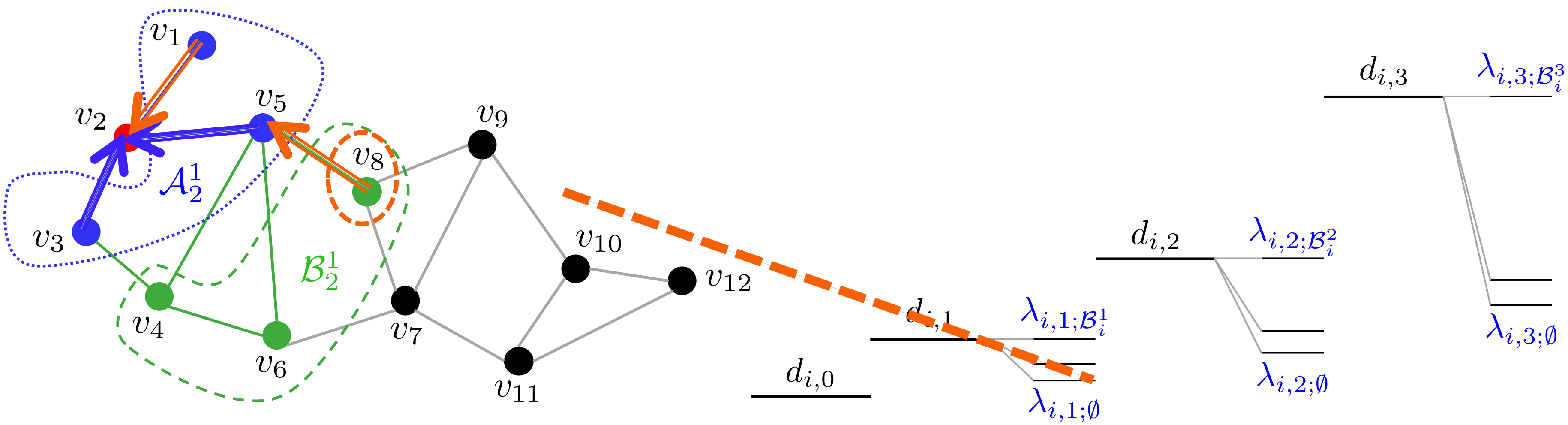
ECC Hierarchy

- **Example**
  - Local decoding: $d_{i,0}$



ECC Hierarchy

- **Example**
  - Local decoding: $d_{i,0}$
  - v_2 obtains extra parities from v_1
    - $\lambda_{2,1;\mathcal{W}} = \lambda_{2,1;\emptyset}$, $\mathcal{W} \subseteq \{v_6, v_8\}$



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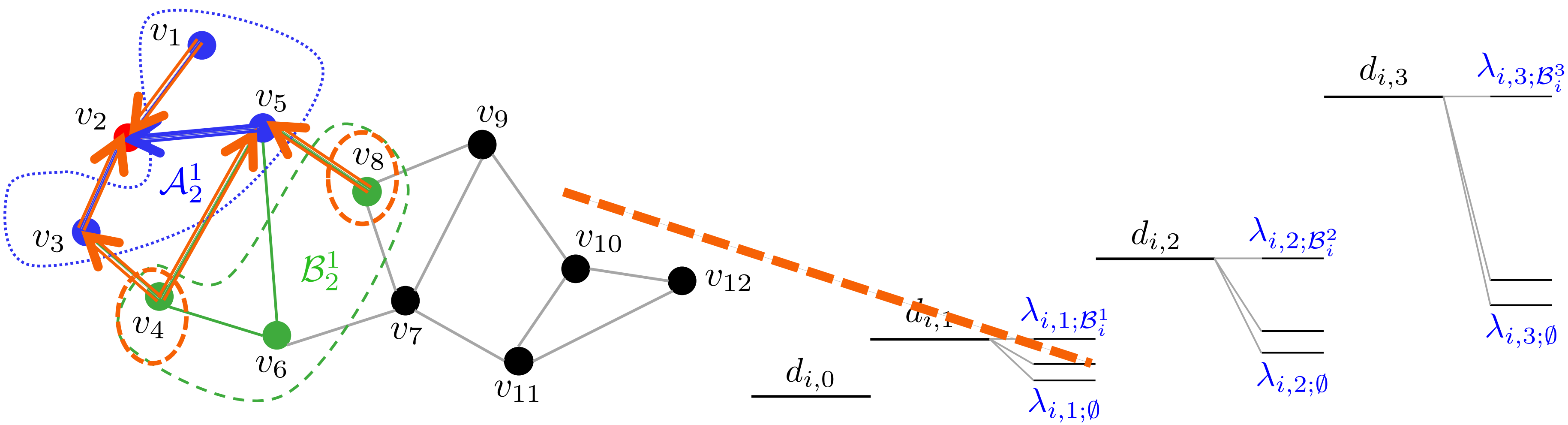
  - v_2 obtains extra parities from v_3 if v_4 is also recovered
    - $\lambda_{2,1;\mathcal{W}} = \lambda_{2,1;\{v_4\}}$, $\{v_4\} \subseteq \mathcal{W} \subset \{v_4, v_6, v_8\}$



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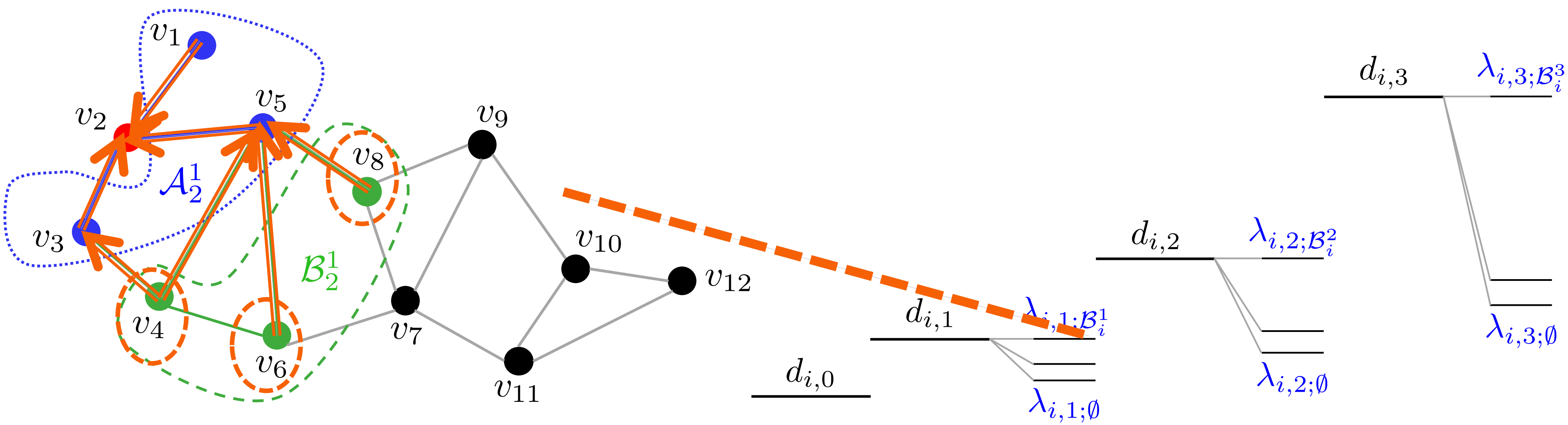
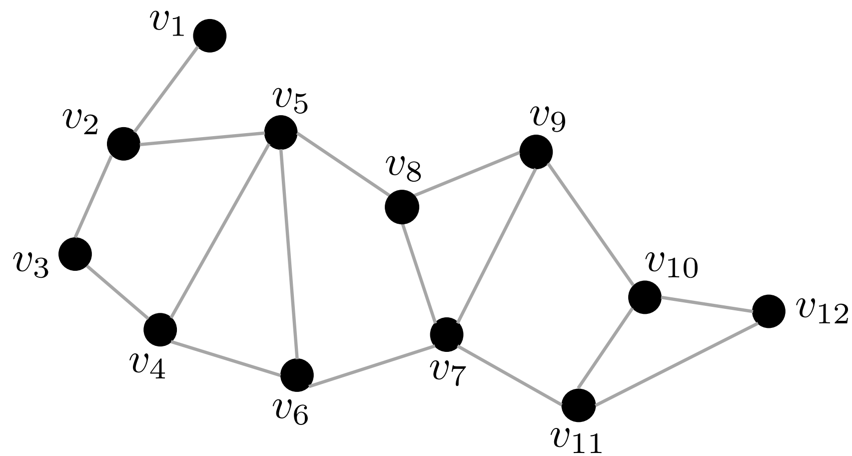
    - $\lambda_{2,1;\mathcal{W}} = \lambda_{2,1;\{v_4\}}$, $\{v_4\} \subseteq \mathcal{W} \subset \{v_4, v_6, v_8\}$
  - v_2 obtains extra parities from v_5 if $\{v_4, v_6, v_8\} \subseteq \mathcal{W}$
    - $\lambda_{2,1;\mathcal{W}} = \lambda_{2,1;\{v_4,v_6,v_8\}} = d_{2,1}$, $\mathcal{W} = \{v_4, v_6, v_8\}$



Example: Single-Level Cooperation

- **Parity part of the generator matrix of a single-level accessible code based on CRS codes**

$\mathbf{A}_{1,1}$	$\mathbf{B}_{1,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{B}_{2,1}\mathbf{U}_1$	$\mathbf{A}_{2,2}$	$\mathbf{B}_{2,3}\mathbf{U}_3$	$\mathbf{0}$	$\mathbf{B}_{2,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{3,2}\mathbf{U}_2$	$\mathbf{A}_{3,3}$	$\mathbf{B}_{3,4}\mathbf{U}_4$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{4,3}\mathbf{U}_3$	$\mathbf{A}_{4,4}$	$\mathbf{B}_{4,5}\mathbf{U}_5$	$\mathbf{B}_{4,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{5,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{B}_{5,4}\mathbf{U}_4$	$\mathbf{A}_{5,5}$	$\mathbf{B}_{5,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{B}_{5,8}\mathbf{U}_8$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{6,4}\mathbf{U}_4$	$\mathbf{B}_{6,5}\mathbf{U}_5$	$\mathbf{A}_{6,6}$	$\mathbf{B}_{6,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{7,6}\mathbf{U}_6$	$\mathbf{A}_{7,7}$	$\mathbf{B}_{7,8}\mathbf{U}_8$	$\mathbf{B}_{7,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{B}_{7,11}\mathbf{U}_{11}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{8,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{B}_{8,7}\mathbf{U}_7$	$\mathbf{A}_{8,8}$	$\mathbf{B}_{8,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{9,7}\mathbf{U}_7$	$\mathbf{B}_{9,8}\mathbf{U}_8$	$\mathbf{A}_{9,9}$	$\mathbf{B}_{9,10}\mathbf{U}_{10}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{10,9}\mathbf{U}_9$	$\mathbf{A}_{10,10}$	$\mathbf{B}_{10,11}\mathbf{U}_{11}$	$\mathbf{B}_{10,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,10}\mathbf{U}_{10}$	$\mathbf{A}_{11,11}$	$\mathbf{B}_{11,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{12,10}\mathbf{U}_{10}$	$\mathbf{B}_{12,11}\mathbf{U}_{11}$	$\mathbf{A}_{12,12}$

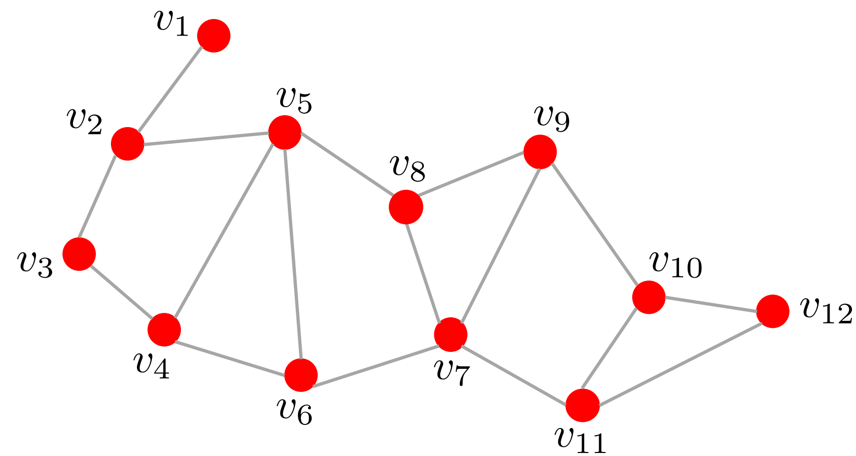


Example: Single-Level Cooperation

- **Parity part of the generator matrix of a single-level accessible code based on CRS codes**

$\boxed{\mathbf{A}_{1,1}}$	$\mathbf{B}_{1,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{B}_{2,1}\mathbf{U}_1$	$\boxed{\mathbf{A}_{2,2}}$	$\mathbf{B}_{2,3}\mathbf{U}_3$	$\mathbf{0}$	$\mathbf{B}_{2,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{3,2}\mathbf{U}_2$	$\boxed{\mathbf{A}_{3,3}}$	$\mathbf{B}_{3,4}\mathbf{U}_4$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{4,3}\mathbf{U}_3$	$\boxed{\mathbf{A}_{4,4}}$	$\mathbf{B}_{4,5}\mathbf{U}_5$	$\mathbf{B}_{4,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{5,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{B}_{5,4}\mathbf{U}_4$	$\boxed{\mathbf{A}_{5,5}}$	$\mathbf{B}_{5,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{B}_{5,8}\mathbf{U}_8$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{6,4}\mathbf{U}_4$	$\mathbf{B}_{6,5}\mathbf{U}_5$	$\boxed{\mathbf{A}_{6,6}}$	$\mathbf{B}_{6,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{7,6}\mathbf{U}_6$	$\boxed{\mathbf{A}_{7,7}}$	$\mathbf{B}_{7,8}\mathbf{U}_8$	$\mathbf{B}_{7,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{B}_{7,11}\mathbf{U}_{11}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{8,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{B}_{8,7}\mathbf{U}_7$	$\boxed{\mathbf{A}_{8,8}}$	$\mathbf{B}_{8,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{9,7}\mathbf{U}_7$	$\mathbf{B}_{9,8}\mathbf{U}_8$	$\boxed{\mathbf{A}_{9,9}}$	$\mathbf{B}_{9,10}\mathbf{U}_{10}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{10,9}\mathbf{U}_9$	$\boxed{\mathbf{A}_{10,10}}$	$\mathbf{B}_{10,11}\mathbf{U}_{11}$	$\mathbf{B}_{10,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,10}\mathbf{U}_{10}$	$\boxed{\mathbf{A}_{11,11}}$	$\mathbf{B}_{11,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{12,10}\mathbf{U}_{10}$	$\mathbf{B}_{12,11}\mathbf{U}_{11}$	$\boxed{\mathbf{A}_{12,12}}$

  - Diagonal components: $\mathbf{A}_{i,i}$



Example: Single-Level Cooperation

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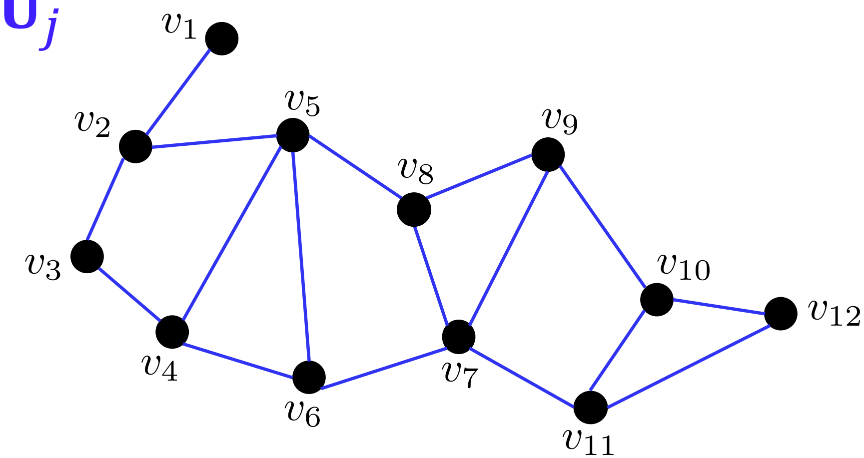
$$
\begin{array}{cccccccccccc}
\mathbf{A}_{1,1} & \boxed{\mathbf{B}_{1,2}\mathbf{U}_2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\boxed{\mathbf{B}_{2,1}\mathbf{U}_1} & \mathbf{A}_{2,2} & \boxed{\mathbf{B}_{2,3}\mathbf{U}_3} & \mathbf{0} & \boxed{\mathbf{B}_{2,5}\mathbf{U}_5} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \boxed{\mathbf{B}_{3,2}\mathbf{U}_2} & \mathbf{A}_{3,3} & \boxed{\mathbf{B}_{3,4}\mathbf{U}_4} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \boxed{\mathbf{B}_{4,3}\mathbf{U}_3} & \mathbf{A}_{4,4} & \boxed{\mathbf{B}_{4,5}\mathbf{U}_5} & \boxed{\mathbf{B}_{4,6}\mathbf{U}_6} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \boxed{\mathbf{B}_{5,2}\mathbf{U}_2} & \mathbf{0} & \boxed{\mathbf{B}_{5,4}\mathbf{U}_4} & \mathbf{A}_{5,5} & \boxed{\mathbf{B}_{5,6}\mathbf{U}_6} & \mathbf{0} & \boxed{\mathbf{B}_{5,8}\mathbf{U}_8} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \boxed{\mathbf{B}_{6,4}\mathbf{U}_4} & \boxed{\mathbf{B}_{6,5}\mathbf{U}_5} & \mathbf{A}_{6,6} & \boxed{\mathbf{B}_{6,7}\mathbf{U}_7} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boxed{\mathbf{B}_{7,6}\mathbf{U}_6} & \mathbf{A}_{7,7} & \boxed{\mathbf{B}_{7,8}\mathbf{U}_8} & \boxed{\mathbf{B}_{7,9}\mathbf{U}_9} & \mathbf{0} & \boxed{\mathbf{B}_{7,11}\mathbf{U}_{11}} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boxed{\mathbf{B}_{8,5}\mathbf{U}_5} & \mathbf{0} & \boxed{\mathbf{B}_{8,7}\mathbf{U}_7} & \mathbf{A}_{8,8} & \boxed{\mathbf{B}_{8,9}\mathbf{U}_9} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boxed{\mathbf{B}_{9,7}\mathbf{U}_7} & \boxed{\mathbf{B}_{9,8}\mathbf{U}_8} & \mathbf{A}_{9,9} & \boxed{\mathbf{B}_{9,10}\mathbf{U}_{10}} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boxed{\mathbf{B}_{10,9}\mathbf{U}_9} & \mathbf{A}_{10,10} & \boxed{\mathbf{B}_{10,11}\mathbf{U}_{11}} & \boxed{\mathbf{B}_{10,12}\mathbf{U}_{12}} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boxed{\mathbf{B}_{11,7}\mathbf{U}_7} & \mathbf{0} & \mathbf{0} & \boxed{\mathbf{B}_{11,10}\mathbf{U}_{10}} & \mathbf{A}_{11,11} & \boxed{\mathbf{B}_{11,12}\mathbf{U}_{12}} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boxed{\mathbf{B}_{12,10}\mathbf{U}_{10}} & \boxed{\mathbf{B}_{12,11}\mathbf{U}_{11}} & \mathbf{A}_{12,12}
\end{array}
$$

  - Diagonal components: $\mathbf{A}_{i,i}$
  - Non-diagonal components: $\mathbf{B}_{i,j}\mathbf{U}_j$



Example: Single-Level Cooperation

- **Parity part of the generator matrix of a single-level accessible code based on CRS codes**

$\mathbf{A}_{1,1}$	$\mathbf{B}_{1,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{B}_{2,1}\mathbf{U}_1$	$\mathbf{A}_{2,2}$	$\mathbf{B}_{2,3}\mathbf{U}_3$	$\mathbf{0}$	$\mathbf{B}_{2,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{3,2}\mathbf{U}_2$	$\mathbf{A}_{3,3}$	$\mathbf{B}_{3,4}\mathbf{U}_4$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{4,3}\mathbf{U}_3$	$\mathbf{A}_{4,4}$	$\mathbf{B}_{4,5}\mathbf{U}_5$	$\mathbf{B}_{4,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{5,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{B}_{5,4}\mathbf{U}_4$	$\mathbf{A}_{5,5}$	$\mathbf{B}_{5,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{B}_{5,8}\mathbf{U}_8$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{6,4}\mathbf{U}_4$	$\mathbf{B}_{6,5}\mathbf{U}_5$	$\mathbf{A}_{6,6}$	$\mathbf{B}_{6,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{7,6}\mathbf{U}_6$	$\mathbf{A}_{7,7}$	$\mathbf{B}_{7,8}\mathbf{U}_8$	$\mathbf{B}_{7,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{B}_{7,11}\mathbf{U}_{11}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{8,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{B}_{8,7}\mathbf{U}_7$	$\mathbf{A}_{8,8}$	$\mathbf{B}_{8,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{9,7}\mathbf{U}_7$	$\mathbf{B}_{9,8}\mathbf{U}_8$	$\mathbf{A}_{9,9}$	$\mathbf{B}_{9,10}\mathbf{U}_{10}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{10,9}\mathbf{U}_9$	$\mathbf{A}_{10,10}$	$\mathbf{B}_{10,11}\mathbf{U}_{11}$	$\mathbf{B}_{10,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,10}\mathbf{U}_{10}$	$\mathbf{A}_{11,11}$	$\mathbf{B}_{11,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{12,10}\mathbf{U}_{10}$	$\mathbf{B}_{12,11}\mathbf{U}_{11}$	$\mathbf{A}_{12,12}$

- Diagonal components: $\mathbf{A}_{i,i}$ $(k_i \times r_i)$
- Non-diagonal components: $\mathbf{B}_{i,j}\mathbf{U}_j$ $(k_i \times \delta_j, \delta_j \times r_j)$
- Components are parts of Cauchy matrices $\mathbf{T}_i$

$$\mathbf{T}_i = \left[\begin{array}{c|c|c|c} \mathbf{A}_{i,i} & \mathbf{B}_{i,j_1} & \dots & \mathbf{B}_{i,j_{|\mathcal{M}_i|}} \\ \hline \mathbf{U}_i & \multicolumn{3}{c}{\mathbf{Z}_i} \end{array}\right]$$

  - $\mathcal{M}_i$: nodes cooperating with v_i in the 1-st level cooperation

$$d_{i,0} = r_i - \delta_i,\ d_{i,1} = r_i + \sum_{v_j \in \mathcal{M}_i} \delta_j\ ,\ \lambda_{i,1;\mathcal{W}} = r_i + \sum_{\mathcal{M}_j \setminus \{v_i\} \subseteq \mathcal{M}_i \cup \mathcal{W}} \delta_j$$

Cauchy Matrices: Scalability and Flexibility

- **Component matrices**

$$\mathbf{T}_i = \left[\begin{array}{c|c|c|c} \mathbf{A}_{i,i} & \mathbf{B}_{i,j_1} & \ldots & \mathbf{B}_{i,j_{|\mathcal{M}_i|}} \\ \hline \mathbf{U}_i & \multicolumn{3}{c}{\mathbf{Z}_i} \end{array}\right]$$

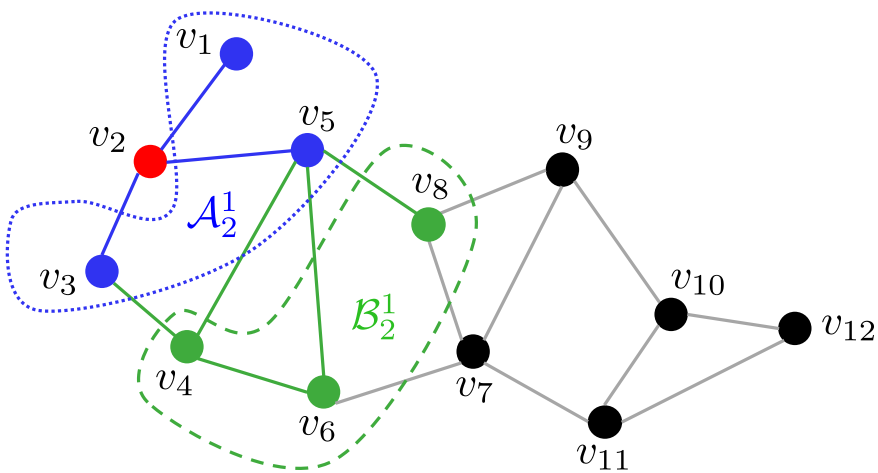
$$\boxed{d_{i,0} = r_i - \delta_i,\ d_{i,1} = r_i + \textstyle\sum_{v_j \in \mathcal{M}_i} \delta_j\ ,\ \lambda_{i,1;\mathcal{W}} = r_i + \textstyle\sum_{\mathcal{M}_j \setminus \{v_i\} \subseteq \mathcal{M}_i \cup \mathcal{W}} \delta_j}$$

- **Why Cauchy matrices?**
  - Component matrices at different nodes can be chosen **independently** – facilitates scalability
  - Cauchy-like structure is inherited through concatenation and splitting of Cauchy matrices
    - Concatenating with a Cauchy matrix of dimension k corresponds to adding k extra parities
    - Splitting the component matrices corresponds to splitting a local cloud -- facilitates flexibility

Example: Hierarchical Erasure Correction

$\mathbf{A}_{1,1}$	$\mathbf{B}_{1,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{B}_{2,1}\mathbf{U}_1$	$\mathbf{A}_{2,2}$	$\mathbf{B}_{2,3}\mathbf{U}_3$	$\mathbf{0}$	$\mathbf{B}_{2,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{3,2}\mathbf{U}_2$	$\mathbf{A}_{3,3}$	$\mathbf{B}_{3,4}\mathbf{U}_4$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{4,3}\mathbf{U}_3$	$\mathbf{A}_{4,4}$	$\mathbf{B}_{4,5}\mathbf{U}_5$	$\mathbf{B}_{4,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{5,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{B}_{5,4}\mathbf{U}_4$	$\mathbf{A}_{5,5}$	$\mathbf{B}_{5,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{B}_{5,8}\mathbf{U}_8$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{6,4}\mathbf{U}_4$	$\mathbf{B}_{6,5}\mathbf{U}_5$	$\mathbf{A}_{6,6}$	$\mathbf{B}_{6,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{7,6}\mathbf{U}_6$	$\mathbf{A}_{7,7}$	$\mathbf{B}_{7,8}\mathbf{U}_8$	$\mathbf{B}_{7,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{B}_{7,11}\mathbf{U}_{11}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{8,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{B}_{8,7}\mathbf{U}_7$	$\mathbf{A}_{8,8}$	$\mathbf{B}_{8,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{9,7}\mathbf{U}_7$	$\mathbf{B}_{9,8}\mathbf{U}_8$	$\mathbf{A}_{9,9}$	$\mathbf{B}_{9,10}\mathbf{U}_{10}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{10,9}\mathbf{U}_9$	$\mathbf{A}_{10,10}$	$\mathbf{B}_{10,11}\mathbf{U}_{11}$	$\mathbf{B}_{10,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,10}\mathbf{U}_{10}$	$\mathbf{A}_{11,11}$	$\mathbf{B}_{11,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{12,10}\mathbf{U}_{10}$	$\mathbf{B}_{12,11}\mathbf{U}_{11}$	$\mathbf{A}_{12,12}$

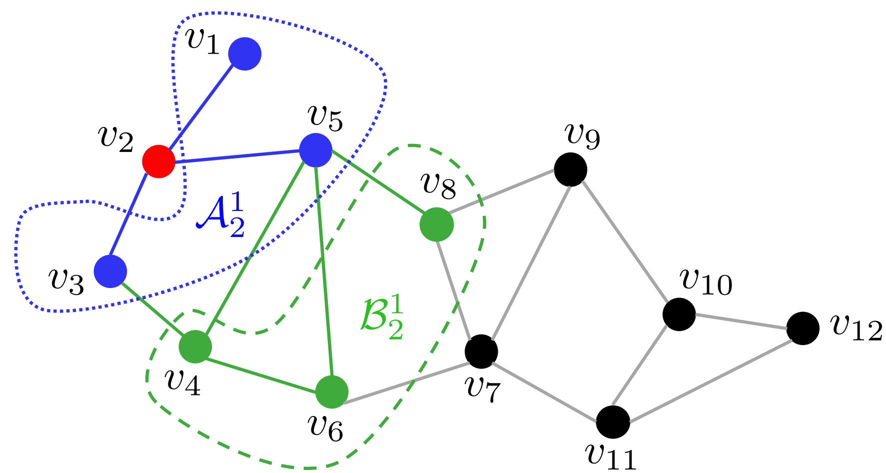
- **ECC hierarchy at node v_2**
  - Local decoding: $d_{2,0} = r_2 - \delta_2$



Example: Hierarchical Erasure Correction

$\mathbf{A}_{1,1}$	$\boxed{\mathbf{B}_{1,2}\mathbf{U}_2}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{B}_{2,1}\mathbf{U}_1$	$\mathbf{A}_{2,2}$	$\mathbf{B}_{2,3}\mathbf{U}_3$	$\mathbf{0}$	$\mathbf{B}_{2,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\boxed{\mathbf{B}_{3,2}\mathbf{U}_2}$	$\mathbf{A}_{3,3}$	$\mathbf{B}_{3,4}\mathbf{U}_4$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{4,3}\mathbf{U}_3$	$\mathbf{A}_{4,4}$	$\mathbf{B}_{4,5}\mathbf{U}_5$	$\mathbf{B}_{4,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\boxed{\mathbf{B}_{5,2}\mathbf{U}_2}$	$\mathbf{0}$	$\mathbf{B}_{5,4}\mathbf{U}_4$	$\mathbf{A}_{5,5}$	$\mathbf{B}_{5,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{B}_{5,8}\mathbf{U}_8$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{6,4}\mathbf{U}_4$	$\mathbf{B}_{6,5}\mathbf{U}_5$	$\mathbf{A}_{6,6}$	$\mathbf{B}_{6,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{7,6}\mathbf{U}_6$	$\mathbf{A}_{7,7}$	$\mathbf{B}_{7,8}\mathbf{U}_8$	$\mathbf{B}_{7,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{B}_{7,11}\mathbf{U}_{11}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{8,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{B}_{8,7}\mathbf{U}_7$	$\mathbf{A}_{8,8}$	$\mathbf{B}_{8,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{9,7}\mathbf{U}_7$	$\mathbf{B}_{9,8}\mathbf{U}_8$	$\mathbf{A}_{9,9}$	$\mathbf{B}_{9,10}\mathbf{U}_{10}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{10,9}\mathbf{U}_9$	$\mathbf{A}_{10,10}$	$\mathbf{B}_{10,11}\mathbf{U}_{11}$	$\mathbf{B}_{10,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,10}\mathbf{U}_{10}$	$\mathbf{A}_{11,11}$	$\mathbf{B}_{11,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{12,10}\mathbf{U}_{10}$	$\mathbf{B}_{12,11}\mathbf{U}_{11}$	$\mathbf{A}_{12,12}$

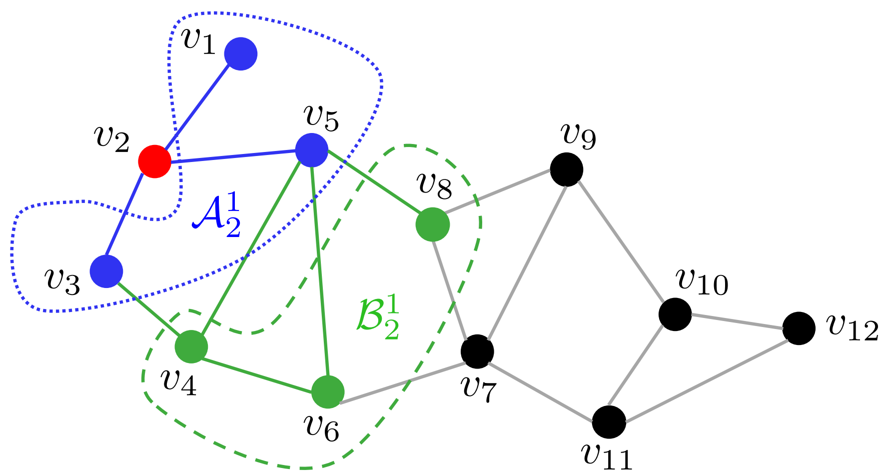
- **ECC hierarchy at node v_2**
  - Local decoding: $d_{2,0} = r_2 - \delta_2$
  - Cross parities at node v_2 can be subtracted from its local parity part if v_1, v_3, v_5 are recovered



Example: Hierarchical Erasure Correction

$\mathbf{A}_{1,1}$	$\mathbf{B}_{1,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{B}_{2,1}\mathbf{U}_1$	$\mathbf{A}_{2,2}$	$\mathbf{B}_{2,3}\mathbf{U}_3$	$\mathbf{0}$	$\mathbf{B}_{2,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{3,2}\mathbf{U}_2$	$\mathbf{A}_{3,3}$	$\mathbf{B}_{3,4}\mathbf{U}_4$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{4,3}\mathbf{U}_3$	$\mathbf{A}_{4,4}$	$\mathbf{B}_{4,5}\mathbf{U}_5$	$\mathbf{B}_{4,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{5,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{B}_{5,4}\mathbf{U}_4$	$\mathbf{A}_{5,5}$	$\mathbf{B}_{5,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{B}_{5,8}\mathbf{U}_8$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{6,4}\mathbf{U}_4$	$\mathbf{B}_{6,5}\mathbf{U}_5$	$\mathbf{A}_{6,6}$	$\mathbf{B}_{6,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{7,6}\mathbf{U}_6$	$\mathbf{A}_{7,7}$	$\mathbf{B}_{7,8}\mathbf{U}_8$	$\mathbf{B}_{7,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{B}_{7,11}\mathbf{U}_{11}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{8,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{B}_{8,7}\mathbf{U}_7$	$\mathbf{A}_{8,8}$	$\mathbf{B}_{8,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{9,7}\mathbf{U}_7$	$\mathbf{B}_{9,8}\mathbf{U}_8$	$\mathbf{A}_{9,9}$	$\mathbf{B}_{9,10}\mathbf{U}_{10}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{10,9}\mathbf{U}_9$	$\mathbf{A}_{10,10}$	$\mathbf{B}_{10,11}\mathbf{U}_{11}$	$\mathbf{B}_{10,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,10}\mathbf{U}_{10}$	$\mathbf{A}_{11,11}$	$\mathbf{B}_{11,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{12,10}\mathbf{U}_{10}$	$\mathbf{B}_{12,11}\mathbf{U}_{11}$	$\mathbf{A}_{12,12}$

- **ECC hierarchy at node v_2**
  - Local decoding: $d_{2,0} = r_2 - \delta_2$
  - Cross parities at node v_2 can be subtracted from its local parity part if v_1, v_3, v_5 are recovered
  - Additional cross parities of $\mathbf{c}_2$ can be obtained from v_j if all other neighbors of v_j except for v_2 are recovered:
    - $\lambda_{2,1;\mathcal{W}} = r_2 + \sum_{\mathcal{M}_j \setminus \{v_2\} \subseteq \mathcal{M}_i \cup \mathcal{W}} \delta_j$



Example: Fast Recovery Speed

$\mathbf{A}_{1,1}$	$\mathbf{B}_{1,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{B}_{2,1}\mathbf{U}_1$	$\mathbf{A}_{2,2}$	$\mathbf{B}_{2,3}\mathbf{U}_3$	$\mathbf{0}$	$\mathbf{B}_{2,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{3,2}\mathbf{U}_2$	$\mathbf{A}_{3,3}$	$\mathbf{B}_{3,4}\mathbf{U}_4$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{4,3}\mathbf{U}_3$	$\mathbf{A}_{4,4}$	$\mathbf{B}_{4,5}\mathbf{U}_5$	$\mathbf{B}_{4,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{5,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{B}_{5,4}\mathbf{U}_4$	$\mathbf{A}_{5,5}$	$\mathbf{B}_{5,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{B}_{5,8}\mathbf{U}_8$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{6,4}\mathbf{U}_4$	$\mathbf{B}_{6,5}\mathbf{U}_5$	$\mathbf{A}_{6,6}$	$\mathbf{B}_{6,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{7,6}\mathbf{U}_6$	$\mathbf{A}_{7,7}$	$\mathbf{B}_{7,8}\mathbf{U}_8$	$\mathbf{B}_{7,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{B}_{7,11}\mathbf{U}_{11}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{8,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{B}_{8,7}\mathbf{U}_7$	$\mathbf{A}_{8,8}$	$\mathbf{B}_{8,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{9,7}\mathbf{U}_7$	$\mathbf{B}_{9,8}\mathbf{U}_8$	$\mathbf{A}_{9,9}$	$\mathbf{B}_{9,10}\mathbf{U}_{10}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{10,9}\mathbf{U}_9$	$\mathbf{A}_{10,10}$	$\mathbf{B}_{10,11}\mathbf{U}_{11}$	$\mathbf{B}_{10,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,10}\mathbf{U}_{10}$	$\mathbf{A}_{11,11}$	$\mathbf{B}_{11,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{12,10}\mathbf{U}_{10}$	$\mathbf{B}_{12,11}\mathbf{U}_{11}$	$\mathbf{A}_{12,12}$

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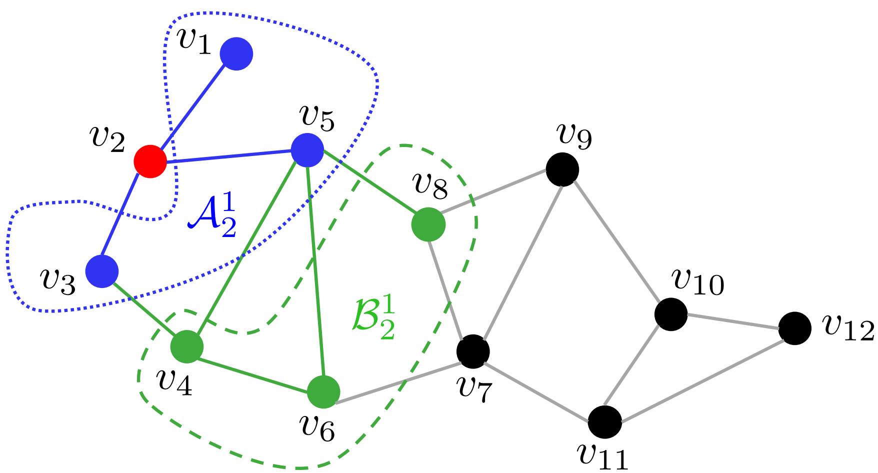

Example: Fast Recovery Speed

$\mathbf{A}_{1,1}$	$\boxed{\mathbf{B}_{1,2}\mathbf{U}_2}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\boxed{\mathbf{B}_{2,1}\mathbf{U}_1}$	$\mathbf{A}_{2,2}$	$\boxed{\mathbf{B}_{2,3}\mathbf{U}_3}$	$\mathbf{0}$	$\boxed{\mathbf{B}_{2,5}\mathbf{U}_5}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\boxed{\mathbf{B}_{3,2}\mathbf{U}_2}$	$\mathbf{A}_{3,3}$	$\mathbf{B}_{3,4}\mathbf{U}_4$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\boxed{\mathbf{B}_{4,3}\mathbf{U}_3}$	$\mathbf{A}_{4,4}$	$\boxed{\mathbf{B}_{4,5}\mathbf{U}_5}$	$\mathbf{B}_{4,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\boxed{\mathbf{B}_{5,2}\mathbf{U}_2}$	$\mathbf{0}$	$\mathbf{B}_{5,4}\mathbf{U}_4$	$\mathbf{A}_{5,5}$	$\mathbf{B}_{5,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{B}_{5,8}\mathbf{U}_8$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{6,4}\mathbf{U}_4$	$\boxed{\mathbf{B}_{6,5}\mathbf{U}_5}$	$\mathbf{A}_{6,6}$	$\mathbf{B}_{6,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{7,6}\mathbf{U}_6$	$\mathbf{A}_{7,7}$	$\mathbf{B}_{7,8}\mathbf{U}_8$	$\mathbf{B}_{7,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{B}_{7,11}\mathbf{U}_{11}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\boxed{\mathbf{B}_{8,5}\mathbf{U}_5}$	$\mathbf{0}$	$\mathbf{B}_{8,7}\mathbf{U}_7$	$\mathbf{A}_{8,8}$	$\mathbf{B}_{8,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{9,7}\mathbf{U}_7$	$\mathbf{B}_{9,8}\mathbf{U}_8$	$\mathbf{A}_{9,9}$	$\mathbf{B}_{9,10}\mathbf{U}_{10}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{10,9}\mathbf{U}_9$	$\mathbf{A}_{10,10}$	$\mathbf{B}_{10,11}\mathbf{U}_{11}$	$\mathbf{B}_{10,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,10}\mathbf{U}_{10}$	$\mathbf{A}_{11,11}$	$\mathbf{B}_{11,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{12,10}\mathbf{U}_{10}$	$\mathbf{B}_{12,11}\mathbf{U}_{11}$	$\mathbf{A}_{12,12}$

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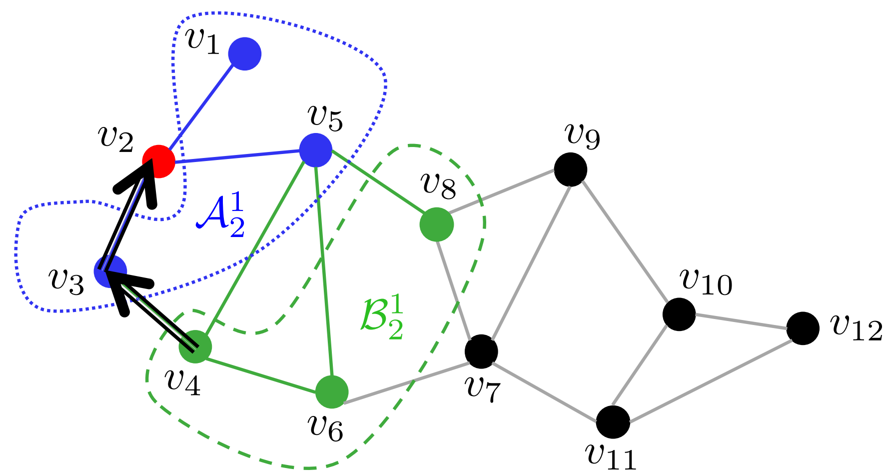

Example: Fast Recovery Speed

$A_{1,1}$	$\boxed{B_{1,2}U_2}$	0	0	0	0	0	0	0	0	0	0
$\boxed{B_{2,1}U_1}$	$\boxed{A_{2,2}}$	$\boxed{B_{2,3}U_3}$	0	$\boxed{B_{2,5}U_5}$	0	0	0	0	0	0	0
0	$\boxed{B_{3,2}U_2}$	$A_{3,3}$	$B_{3,4}U_4$	0	0	0	0	0	0	0	0
0	0	$\boxed{B_{4,3}U_3}$	$A_{4,4}$	$\boxed{B_{4,5}U_5}$	$B_{4,6}U_6$	0	0	0	0	0	0
0	$\boxed{B_{5,2}U_2}$	0	$B_{5,4}U_4$	$A_{5,5}$	$B_{5,6}U_6$	0	$B_{5,8}U_8$	0	0	0	0
0	0	0	$B_{6,4}U_4$	$\boxed{B_{6,5}U_5}$	$A_{6,6}$	$B_{6,7}U_7$	0	0	0	0	0
0	0	0	0	0	$B_{7,6}U_6$	$A_{7,7}$	$B_{7,8}U_8$	$B_{7,9}U_9$	0	$B_{7,11}U_{11}$	0
0	0	0	0	$\boxed{B_{8,5}U_5}$	0	$B_{8,7}U_7$	$A_{8,8}$	$B_{8,9}U_9$	0	0	0
0	0	0	0	0	0	$B_{9,7}U_7$	$B_{9,8}U_8$	$A_{9,9}$	$B_{9,10}U_{10}$	0	0
0	0	0	0	0	0	0	0	$B_{10,9}U_9$	$A_{10,10}$	$B_{10,11}U_{11}$	$B_{10,12}U_{12}$
0	0	0	0	0	0	$B_{11,7}U_7$	0	0	$B_{11,10}U_{10}$	$A_{11,11}$	$B_{11,12}U_{12}$
0	0	0	0	0	0	0	0	0	$B_{12,10}U_{10}$	$B_{12,11}U_{11}$	$A_{12,12}$

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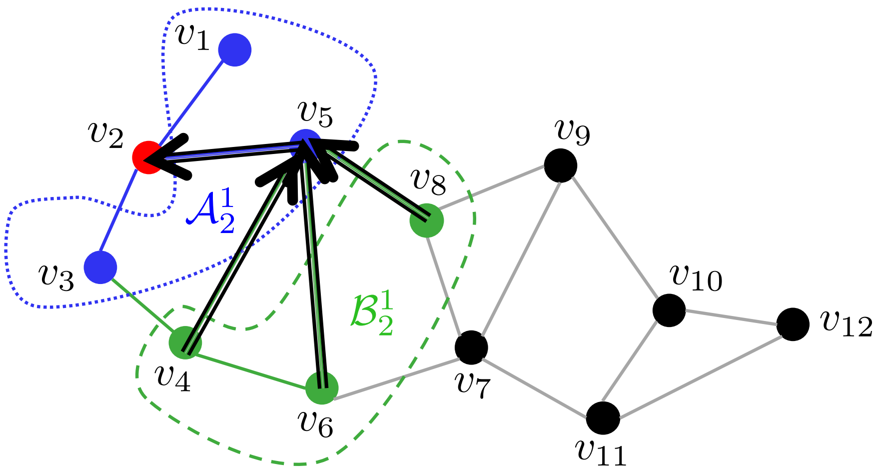

Example: Fast Recovery Speed

$\mathbf{A}_{1,1}$	$\mathbf{B}_{1,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{B}_{2,1}\mathbf{U}_1$	$\mathbf{A}_{2,2}$	$\mathbf{B}_{2,3}\mathbf{U}_3$	$\mathbf{0}$	$\mathbf{B}_{2,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{3,2}\mathbf{U}_2$	$\mathbf{A}_{3,3}$	$\mathbf{B}_{3,4}\mathbf{U}_4$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{4,3}\mathbf{U}_3$	$\mathbf{A}_{4,4}$	$\mathbf{B}_{4,5}\mathbf{U}_5$	$\mathbf{B}_{4,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{5,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{B}_{5,4}\mathbf{U}_4$	$\mathbf{A}_{5,5}$	$\mathbf{B}_{5,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{B}_{5,8}\mathbf{U}_8$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{6,4}\mathbf{U}_4$	$\mathbf{B}_{6,5}\mathbf{U}_5$	$\mathbf{A}_{6,6}$	$\mathbf{B}_{6,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{7,6}\mathbf{U}_6$	$\mathbf{A}_{7,7}$	$\mathbf{B}_{7,8}\mathbf{U}_8$	$\mathbf{B}_{7,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{B}_{7,11}\mathbf{U}_{11}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{8,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{B}_{8,7}\mathbf{U}_7$	$\mathbf{A}_{8,8}$	$\mathbf{B}_{8,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{9,7}\mathbf{U}_7$	$\mathbf{B}_{9,8}\mathbf{U}_8$	$\mathbf{A}_{9,9}$	$\mathbf{B}_{9,10}\mathbf{U}_{10}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{10,9}\mathbf{U}_9$	$\mathbf{A}_{10,10}$	$\mathbf{B}_{10,11}\mathbf{U}_{11}$	$\mathbf{B}_{10,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,10}\mathbf{U}_{10}$	$\mathbf{A}_{11,11}$	$\mathbf{B}_{11,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{12,10}\mathbf{U}_{10}$	$\mathbf{B}_{12,11}\mathbf{U}_{11}$	$\mathbf{A}_{12,12}$

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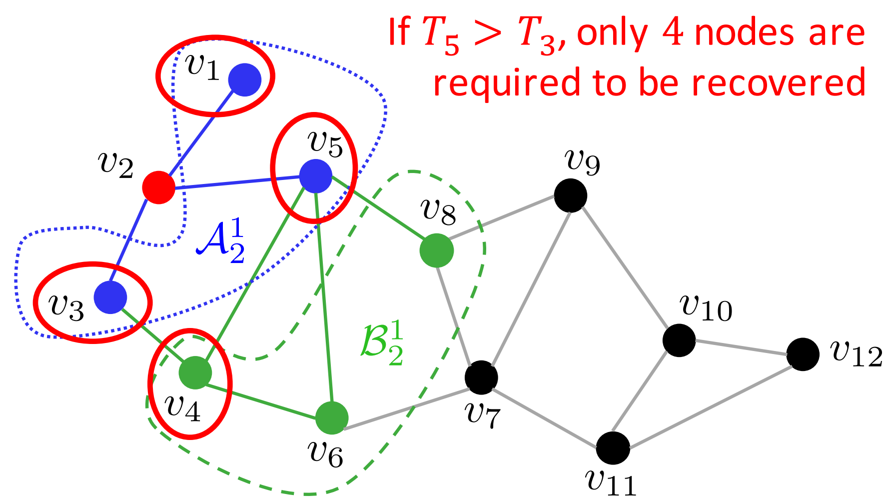

Example: Fast Recovery Speed

$\mathbf{A}_{1,1}$	$\mathbf{B}_{1,2}\mathbf{U}_2$	0	0	0	0	0	0	0	0	0	0
$\mathbf{B}_{2,1}\mathbf{U}_1$	$\mathbf{A}_{2,2}$	$\mathbf{B}_{2,3}\mathbf{U}_3$	0	$\mathbf{B}_{2,5}\mathbf{U}_5$	0	0	0	0	0	0	0
0	$\mathbf{B}_{3,2}\mathbf{U}_2$	$\mathbf{A}_{3,3}$	$\mathbf{B}_{3,4}\mathbf{U}_4$	0	0	0	0	0	0	0	0
0	0	$\mathbf{B}_{4,3}\mathbf{U}_3$	$\mathbf{A}_{4,4}$	$\mathbf{B}_{4,5}\mathbf{U}_5$	$\mathbf{B}_{4,6}\mathbf{U}_6$	0	0	0	0	0	0
0	$\mathbf{B}_{5,2}\mathbf{U}_2$	0	$\mathbf{B}_{5,4}\mathbf{U}_4$	$\mathbf{A}_{5,5}$	$\mathbf{B}_{5,6}\mathbf{U}_6$	0	$\mathbf{B}_{5,8}\mathbf{U}_8$	0	0	0	0
0	0	0	$\mathbf{B}_{6,4}\mathbf{U}_4$	$\mathbf{B}_{6,5}\mathbf{U}_5$	$\mathbf{A}_{6,6}$	$\mathbf{B}_{6,7}\mathbf{U}_7$	0	0	0	0	0
0	0	0	0	0	$\mathbf{B}_{7,6}\mathbf{U}_6$	$\mathbf{A}_{7,7}$	$\mathbf{B}_{7,8}\mathbf{U}_8$	$\mathbf{B}_{7,9}\mathbf{U}_9$	0	$\mathbf{B}_{7,11}\mathbf{U}_{11}$	0
0	0	0	0	$\mathbf{B}_{8,5}\mathbf{U}_5$	0	$\mathbf{B}_{8,7}\mathbf{U}_7$	$\mathbf{A}_{8,8}$	$\mathbf{B}_{8,9}\mathbf{U}_9$	0	0	0
0	0	0	0	0	0	$\mathbf{B}_{9,7}\mathbf{U}_7$	$\mathbf{B}_{9,8}\mathbf{U}_8$	$\mathbf{A}_{9,9}$	$\mathbf{B}_{9,10}\mathbf{U}_{10}$	0	0
0	0	0	0	0	0	0	0	$\mathbf{B}_{10,9}\mathbf{U}_9$	$\mathbf{A}_{10,10}$	$\mathbf{B}_{10,11}\mathbf{U}_{11}$	$\mathbf{B}_{10,12}\mathbf{U}_{12}$
0	0	0	0	0	0	$\mathbf{B}_{11,7}\mathbf{U}_7$	0	0	$\mathbf{B}_{11,10}\mathbf{U}_{10}$	$\mathbf{A}_{11,11}$	$\mathbf{B}_{11,12}\mathbf{U}_{12}$
0	0	0	0	0	0	0	0	0	$\mathbf{B}_{12,10}\mathbf{U}_{10}$	$\mathbf{B}_{12,11}\mathbf{U}_{11}$	$\mathbf{A}_{12,12}$

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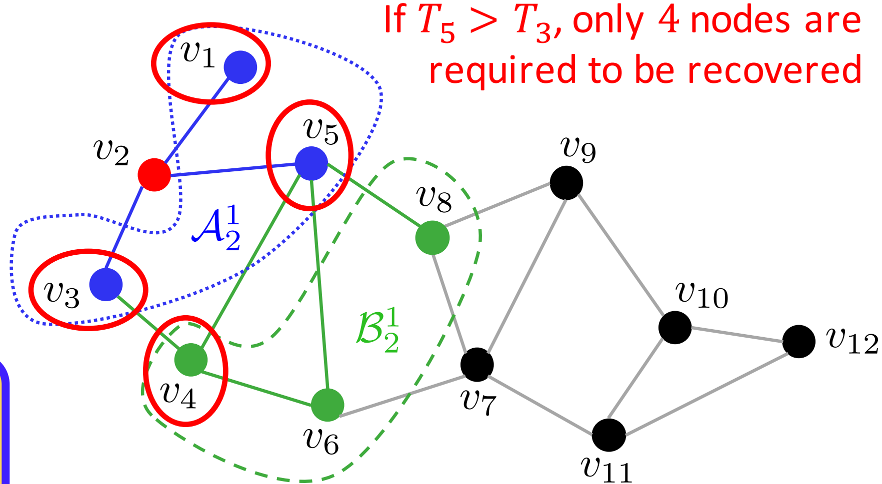
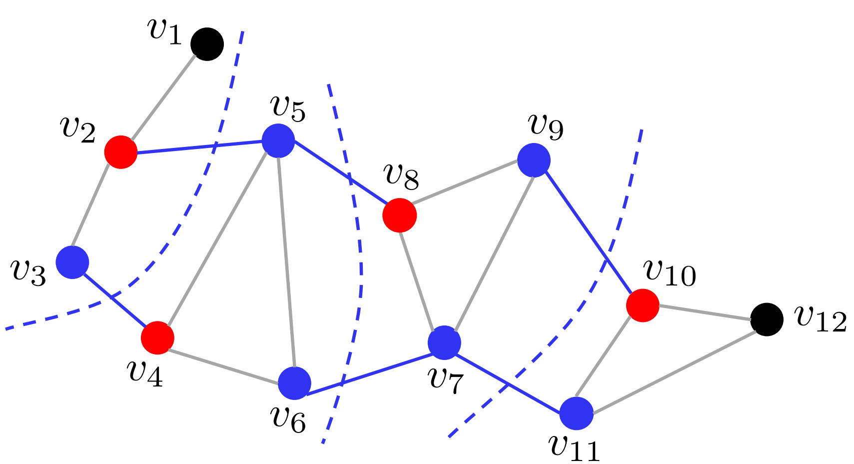
If $T_5 > T_3$, only 4 nodes are required to be recovered



Nodes in the DSN automatically choose the fastest path for recovery

Example: Flexible Erasure Pattern

- **Error pattern:** v_2, v_4, v_8, v_{10} are not locally recoverable
  - Recoverable in topology-aware coding
    - Each red node cooperates with 3 nodes



[1] S. Yang et al., "*Hierarchical coding to enable scalability and flexibility in heterogeneous cloud storage*", IEEE CLOBECOM, 2019

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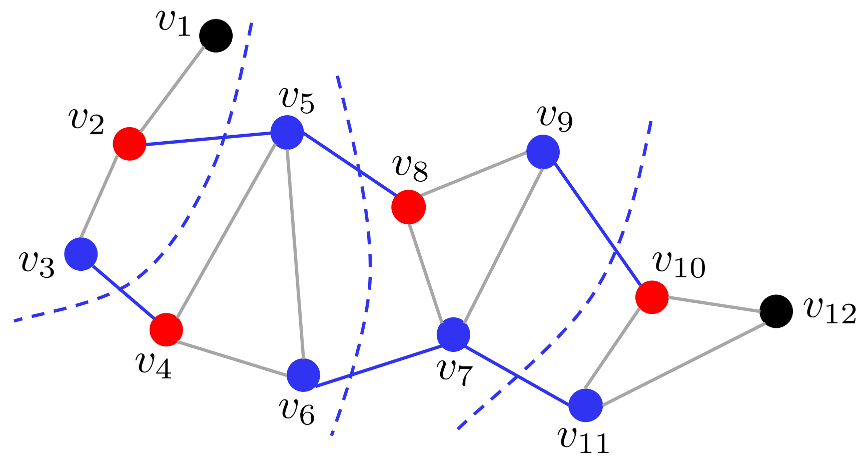
    - Each group contains at most one non-locally recoverable node



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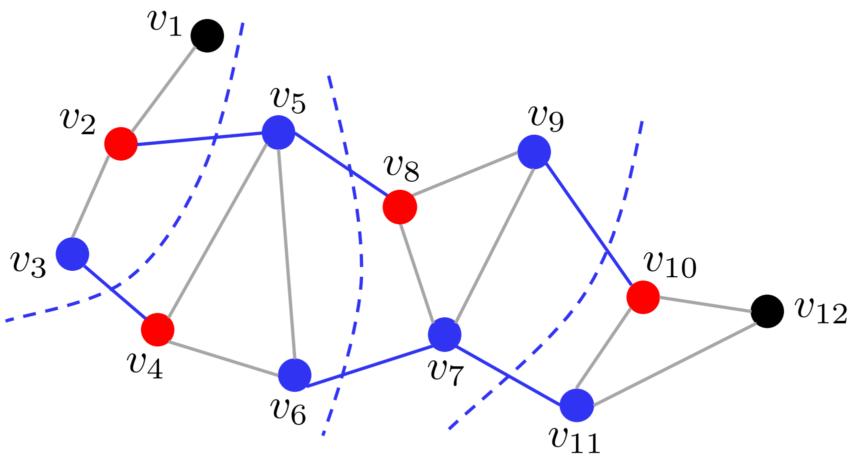
  - The EC capability at each node (except for v_1, v_{12}) decreases
    - information that flows through edges marked in blue no longer exist



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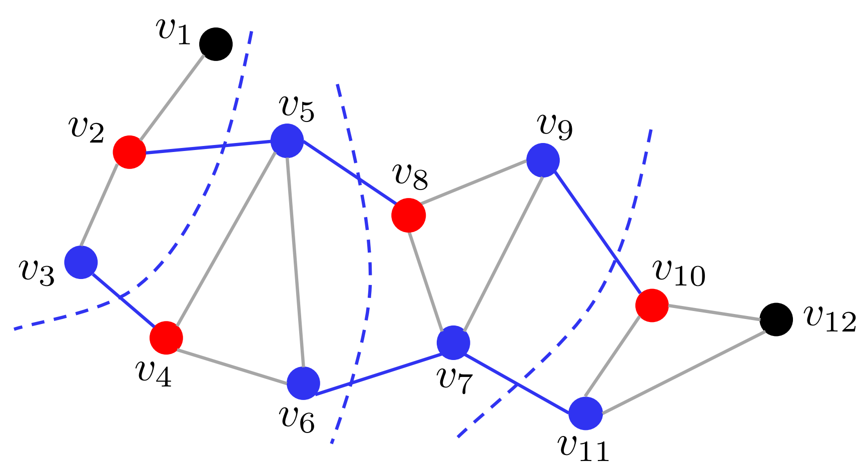
Topology-aware coding enables information flow from more reliable nodes to less reliable nodes, tolerating more flexible erasure patterns



[1] S. Yang et al., "*Hierarchical coding to enable scalability and flexibility in heterogeneous cloud storage*", IEEE CLOBECOM, 2019

Outline

- **Introduction**
  - Motivation and Model
  - Existing work
- **Cooperative Data Protection**
  - ECC hierarchy
  - Single-level cooperation
- **Multi-level cooperation**
  - Cooperation graphs and compatible graphs
  - Construction over compatible graphs
- **Conclusion**

Example: Multi-Level Cooperation

- **Parity part of the generator matrix**

$\mathbf{A}_{1,1}$	$\mathbf{B}_{1,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{B}_{2,1}\mathbf{U}_1$	$\mathbf{A}_{2,2}$	$\mathbf{B}_{2,3}\mathbf{U}_3$	$\mathbf{0}$	$\mathbf{B}_{2,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{3,2}\mathbf{U}_2$	$\mathbf{A}_{3,3}$	$\mathbf{B}_{3,4}\mathbf{U}_4$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{4,3}\mathbf{U}_3$	$\mathbf{A}_{4,4}$	$\mathbf{B}_{4,5}\mathbf{U}_5$	$\mathbf{B}_{4,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{5,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{B}_{5,4}\mathbf{U}_4$	$\mathbf{A}_{5,5}$	$\mathbf{B}_{5,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{B}_{5,8}\mathbf{U}_8$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{6,4}\mathbf{U}_4$	$\mathbf{B}_{6,5}\mathbf{U}_5$	$\mathbf{A}_{6,6}$	$\mathbf{B}_{6,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{7,6}\mathbf{U}_6$	$\mathbf{A}_{7,7}$	$\mathbf{B}_{7,8}\mathbf{U}_8$	$\mathbf{B}_{7,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{B}_{7,11}\mathbf{U}_{11}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{8,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{B}_{8,7}\mathbf{U}_7$	$\mathbf{A}_{8,8}$	$\mathbf{B}_{8,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{9,7}\mathbf{U}_7$	$\mathbf{B}_{9,8}\mathbf{U}_8$	$\mathbf{A}_{9,9}$	$\mathbf{B}_{9,10}\mathbf{U}_{10}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{10,9}\mathbf{U}_9$	$\mathbf{A}_{10,10}$	$\mathbf{B}_{10,11}\mathbf{U}_{11}$	$\mathbf{B}_{10,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,10}\mathbf{U}_{10}$	$\mathbf{A}_{11,11}$	$\mathbf{B}_{11,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{12,10}\mathbf{U}_{10}$	$\mathbf{B}_{12,11}\mathbf{U}_{11}$	$\mathbf{A}_{12,12}$

  - Components corresponding to the 1-st level cooperation

	1	2	3	4	5	6	7	8	9	10	11	12
1	*	1										
2	1	*	1		1							
3		1	*	1								
4			1	*	1	1						
5		1		1	*	1		1				
6				1	1	*	1					
7						1	*	1	1		1	
8					1		1	*	1			
9							1	1	*	1		
10									1	*	1	1
11							1			1	*	1
12										1	1	*

	1	2	3	4	5	6	7	8	9	10	11	12
1	*	1										
2	1	*	1		1							
3		1	*	1								
4			1	*	1	1						
5		1		1	*	1		1				
6				1	1	*	1					
7						1	*	1	1		1	
8					1		1	*	1			
9							1	1	*	1		
10									1	*	1	1
11							1			1	*	1
12										1	1	*

Example: Multi-Level Cooperation

- **Parity part of the generator matrix**

$\mathbf{A}_{1,1}$	$\mathbf{B}_{1,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{B}_{2,1}\mathbf{U}_1$	$\mathbf{A}_{2,2}$	$\mathbf{B}_{2,3}\mathbf{U}_3$	$\mathbf{0}$	$\mathbf{B}_{2,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{3,2}\mathbf{U}_2$	$\mathbf{A}_{3,3}$	$\mathbf{B}_{3,4}\mathbf{U}_4$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{4,3}\mathbf{U}_3$	$\mathbf{A}_{4,4}$	$\mathbf{B}_{4,5}\mathbf{U}_5$	$\mathbf{B}_{4,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{5,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{B}_{5,4}\mathbf{U}_4$	$\mathbf{A}_{5,5}$	$\mathbf{B}_{5,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{B}_{5,8}\mathbf{U}_8$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{6,4}\mathbf{U}_4$	$\mathbf{B}_{6,5}\mathbf{U}_5$	$\mathbf{A}_{6,6}$	$\mathbf{B}_{6,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{7,6}\mathbf{U}_6$	$\mathbf{A}_{7,7}$	$\mathbf{B}_{7,8}\mathbf{U}_8$	$\mathbf{B}_{7,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{B}_{7,11}\mathbf{U}_{11}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{8,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{B}_{8,7}\mathbf{U}_7$	$\mathbf{A}_{8,8}$	$\mathbf{B}_{8,9}\mathbf{U}_9$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{9,7}\mathbf{U}_7$	$\mathbf{B}_{9,8}\mathbf{U}_8$	$\mathbf{A}_{9,9}$	$\mathbf{B}_{9,10}\mathbf{U}_{10}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{10,9}\mathbf{U}_9$	$\mathbf{A}_{10,10}$	$\mathbf{B}_{10,11}\mathbf{U}_{11}$	$\mathbf{B}_{10,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,7}\mathbf{U}_7$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{11,10}\mathbf{U}_{10}$	$\mathbf{A}_{11,11}$	$\mathbf{B}_{11,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{12,10}\mathbf{U}_{10}$	$\mathbf{B}_{12,11}\mathbf{U}_{11}$	$\mathbf{A}_{12,12}$

  - Components corresponding to the 1-st level cooperation
  - Components corresponding to higher level cooperation
    - Can be divided into different groups: each group represents a cycle

	1	2	3	4	5	6	7	8	9	10	11	12
1	*	1										
2	1	*	1		1							
3		1	*	1								
4			1	*	1	1						
5		1		1	*	1		1				
6				1	1	*	1					
7						1	*	1	1		1	
8					1		1	*	1			
9							1	1	*	1		
10									1	*	1	1
11							1			1	*	1
12										1	1	*

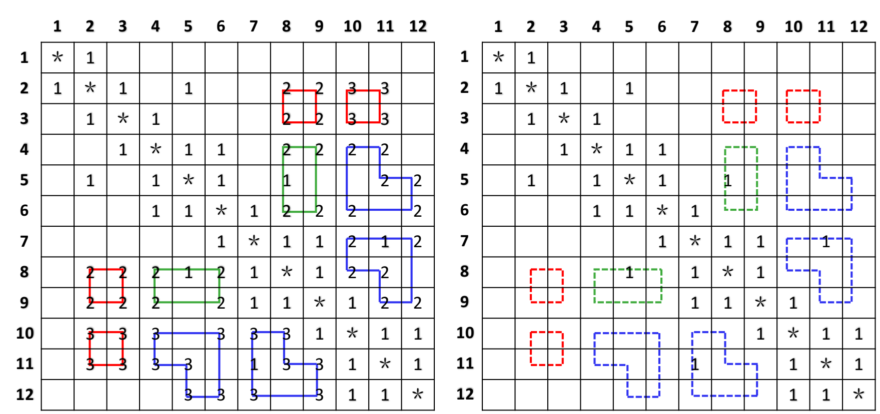
	1	2	3	4	5	6	7	8	9	10	11	12
1	*	1										
2	1	*	1		1							
3		1	*	1								
4			1	*	1	1						
5		1		1	*	1		1				
6				1	1	*	1					
7						1	*	1	1		1	
8					1		1	*	1			
9							1	1	*	1		
10									1	*	1	1
11							1			1	*	1
12										1	1	*

Example: Multi-Level Cooperation

- **Parity part of the generator matrix**

$A_{1,1}$	$B_{1,2}U_2$	0	0	0	0	0	0	0	0	0	0
$B_{2,1}U_1$	$A_{2,2}$	$B_{2,3}U_3$	0	$B_{2,5}U_5$	0	0	$B_cV_{8;2}$	$B_cV_{9;2}$	$B_dV_{10;3}$	$B_dV_{11;3}$	0
0	$B_{3,2}U_2$	$A_{3,3}$	$B_{3,4}U_4$	0	0	0	$B_eV_{8;2}$	$B_eV_{9;2}$	$B_fV_{10;3}$	$B_fV_{11;3}$	0
0	0	$B_{4,3}U_3$	$A_{4,4}$	$B_{4,5}U_5$	$B_{4,6}U_6$	0	$B_\alpha V_{8;2}$	$B_\alpha V_{9;2}$	$B_gV_{10;2}$	$B_gV_{11;2}$	0
0	$B_{5,2}U_2$	0	$B_{5,4}U_4$	$A_{5,5}$	$B_{5,6}U_6$	0	$B_{5,8}U_8$	0	$B_hV_{10;2}$	0	$B_hV_{12;2}$
0	0	0	$B_{6,4}U_4$	$B_{6,5}U_5$	$A_{6,6}$	$B_{6,7}U_7$	$B_\beta V_{8;2}$	$B_\beta V_{9;2}$	$B_jV_{10;2}$	0	$B_jV_{12;2}$
0	0	0	0	0	$B_{7,6}U_6$	$A_{7,7}$	$B_{7,8}U_8$	$B_{7,9}U_9$	$B_lV_{10;2}$	$B_{7,11}U_{11}$	$B_lV_{12;2}$
0	$B_mV_{2;2}$	$B_mV_{3;2}$	$B_yV_{4;2}$	$B_{8,5}U_5$	$B_yV_{6;2}$	$B_{8,7}U_7$	$A_{8,8}$	$B_{8,9}U_9$	$B_nV_{10;2}$	$B_nV_{11;2}$	0
0	$B_oV_{2;2}$	$B_oV_{3;2}$	$B_zV_{4;2}$	0	$B_zV_{6;2}$	$B_{9,7}U_7$	$B_{9,8}U_8$	$A_{9,9}$	$B_{9,10}U_{10}$	$B_pV_{11;2}$	$B_pV_{12;2}$
0	$B_qV_{2;3}$	$B_qV_{3;3}$	$B_rV_{4;2}$	0	$B_rV_{6;3}$	$B_sV_{7;3}$	$B_sV_{8;3}$	$B_{10,9}U_9$	$A_{10,10}$	$B_{10,11}U_{11}$	$B_{10,12}U_{12}$
0	$B_xV_{2;3}$	$B_xV_{3;3}$	$B_tV_{4;2}$	$B_tV_{5;2}$	0	$B_{11,7}U_7$	$B_uV_{8;3}$	$B_uV_{9;3}$	$B_{11,10}U_{10}$	$A_{11,11}$	$B_{11,12}U_{12}$
0	0	0	0	$B_vV_{5;2}$	$B_vV_{6;3}$	$B_wV_{7;3}$	0	$B_wV_{9;3}$	$B_{12,10}U_{10}$	$B_{12,11}U_{11}$	$A_{12,12}$

  - Components corresponding to the 1-st level cooperation
  - Components corresponding to higher level cooperation
    - Can be divided into different groups: each group represents a cycle
    - Nodes on each vertical edge have the same cooperation level

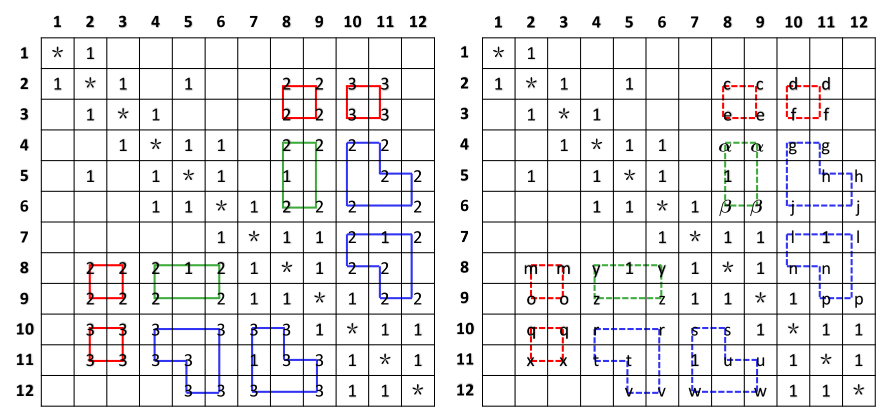


Example: Multi-Level Cooperation

- **Parity part of the generator matrix**

$\mathbf{A}_{1,1}$	$\mathbf{B}_{1,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\mathbf{B}_{2,1}\mathbf{U}_1$	$\mathbf{A}_{2,2}$	$\mathbf{B}_{2,3}\mathbf{U}_3$	$\mathbf{0}$	$\mathbf{B}_{2,5}\mathbf{U}_5$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_c\mathbf{V}_{8;2}$	$\mathbf{B}_c\mathbf{V}_{9;2}$	$\mathbf{B}_d\mathbf{V}_{10;3}$	$\mathbf{B}_d\mathbf{V}_{11;3}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{3,2}\mathbf{U}_2$	$\mathbf{A}_{3,3}$	$\mathbf{B}_{3,4}\mathbf{U}_4$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_e\mathbf{V}_{8;2}$	$\mathbf{B}_e\mathbf{V}_{9;2}$	$\mathbf{B}_f\mathbf{V}_{10;3}$	$\mathbf{B}_f\mathbf{V}_{11;3}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{4,3}\mathbf{U}_3$	$\mathbf{A}_{4,4}$	$\mathbf{B}_{4,5}\mathbf{U}_5$	$\mathbf{B}_{4,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{B}_\alpha\mathbf{V}_{8;2}$	$\mathbf{B}_\alpha\mathbf{V}_{9;2}$	$\mathbf{B}_g\mathbf{V}_{10;2}$	$\mathbf{B}_g\mathbf{V}_{11;2}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_{5,2}\mathbf{U}_2$	$\mathbf{0}$	$\mathbf{B}_{5,4}\mathbf{U}_4$	$\mathbf{A}_{5,5}$	$\mathbf{B}_{5,6}\mathbf{U}_6$	$\mathbf{0}$	$\mathbf{B}_{5,8}\mathbf{U}_8$	$\mathbf{0}$	$\mathbf{B}_h\mathbf{V}_{10;2}$	$\mathbf{0}$	$\mathbf{B}_h\mathbf{V}_{12;2}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{6,4}\mathbf{U}_4$	$\mathbf{B}_{6,5}\mathbf{U}_5$	$\mathbf{A}_{6,6}$	$\mathbf{B}_{6,7}\mathbf{U}_7$	$\mathbf{B}_\beta\mathbf{V}_{8;2}$	$\mathbf{B}_\beta\mathbf{V}_{9;2}$	$\mathbf{B}_j\mathbf{V}_{10;2}$	$\mathbf{0}$	$\mathbf{B}_j\mathbf{V}_{12;2}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_{7,6}\mathbf{U}_6$	$\mathbf{A}_{7,7}$	$\mathbf{B}_{7,8}\mathbf{U}_8$	$\mathbf{B}_{7,9}\mathbf{U}_9$	$\mathbf{B}_l\mathbf{V}_{10;2}$	$\mathbf{B}_{7,11}\mathbf{U}_{11}$	$\mathbf{B}_l\mathbf{V}_{12;2}$
$\mathbf{0}$	$\mathbf{B}_m\mathbf{V}_{2;2}$	$\mathbf{B}_m\mathbf{V}_{3;2}$	$\mathbf{B}_y\mathbf{V}_{4;2}$	$\mathbf{B}_{8,5}\mathbf{U}_5$	$\mathbf{B}_y\mathbf{V}_{6;2}$	$\mathbf{B}_{8,7}\mathbf{U}_7$	$\mathbf{A}_{8,8}$	$\mathbf{B}_{8,9}\mathbf{U}_9$	$\mathbf{B}_n\mathbf{V}_{10;2}$	$\mathbf{B}_n\mathbf{V}_{11;2}$	$\mathbf{0}$
$\mathbf{0}$	$\mathbf{B}_o\mathbf{V}_{2;2}$	$\mathbf{B}_o\mathbf{V}_{3;2}$	$\mathbf{B}_z\mathbf{V}_{4;2}$	$\mathbf{0}$	$\mathbf{B}_z\mathbf{V}_{6;2}$	$\mathbf{B}_{9,7}\mathbf{U}_7$	$\mathbf{B}_{9,8}\mathbf{U}_8$	$\mathbf{A}_{9,9}$	$\mathbf{B}_{9,10}\mathbf{U}_{10}$	$\mathbf{B}_p\mathbf{V}_{11;2}$	$\mathbf{B}_p\mathbf{V}_{12;2}$
$\mathbf{0}$	$\mathbf{B}_q\mathbf{V}_{2;3}$	$\mathbf{B}_q\mathbf{V}_{3;3}$	$\mathbf{B}_r\mathbf{V}_{4;2}$	$\mathbf{0}$	$\mathbf{B}_r\mathbf{V}_{6;3}$	$\mathbf{B}_s\mathbf{V}_{7;3}$	$\mathbf{B}_s\mathbf{V}_{8;3}$	$\mathbf{B}_{10,9}\mathbf{U}_9$	$\mathbf{A}_{10,10}$	$\mathbf{B}_{10,11}\mathbf{U}_{11}$	$\mathbf{B}_{10,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{B}_x\mathbf{V}_{2;3}$	$\mathbf{B}_x\mathbf{V}_{3;3}$	$\mathbf{B}_t\mathbf{V}_{4;2}$	$\mathbf{B}_t\mathbf{V}_{5;2}$	$\mathbf{0}$	$\mathbf{B}_{11,7}\mathbf{U}_7$	$\mathbf{B}_u\mathbf{V}_{8;3}$	$\mathbf{B}_u\mathbf{V}_{9;3}$	$\mathbf{B}_{11,10}\mathbf{U}_{10}$	$\mathbf{A}_{11,11}$	$\mathbf{B}_{11,12}\mathbf{U}_{12}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{B}_v\mathbf{V}_{5;2}$	$\mathbf{B}_v\mathbf{V}_{6;3}$	$\mathbf{B}_w\mathbf{V}_{7;3}$	$\mathbf{0}$	$\mathbf{B}_w\mathbf{V}_{9;3}$	$\mathbf{B}_{12,10}\mathbf{U}_{10}$	$\mathbf{B}_{12,11}\mathbf{U}_{11}$	$\mathbf{A}_{12,12}$

  - Components corresponding to the 1-st level cooperation
  - Components corresponding to higher level cooperation
    - Can be divided into different groups: each group represents a cycle
    - Nodes on each vertical edge have the same cooperation level
    - Nodes on each horizontal edge has the same component matrix that transfer a message into a vector of cross parities



Cooperation Graphs

- **The aforementioned matrix is referred to as a *cooperation matrix***
  - Construct the cooperation graph from the cooperation matrix
    - Each cycle represents a pair of edges/triangles, with an arrow pointing from one to the other, labelled with the cooperation level

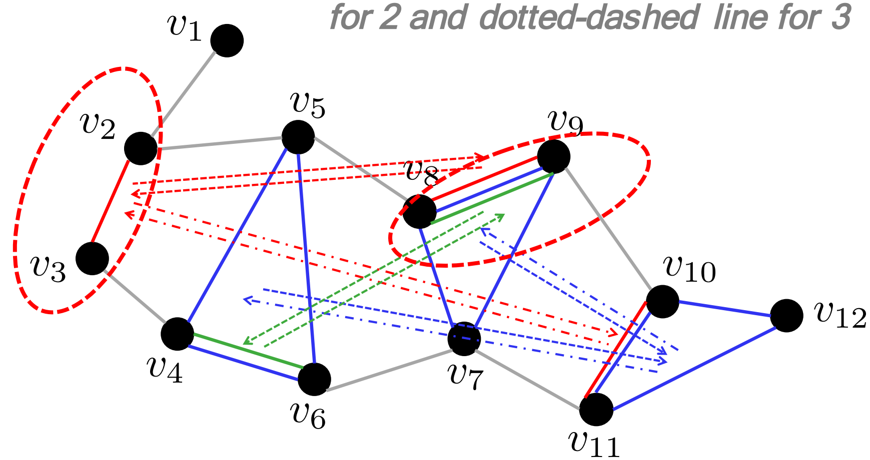
Y_1

	1	2	3	4	5	6	7	8	9	10	11	12
1	*	1										
2	1	*	1		1			2	2	3	3	
3		1	*	1				2	2	3	3	
4			1	*	1	1		2	2	2	2	
5		1		1	*	1		1			2	2
6				1	1	*	1	2	2	2		2
7						1	*	1	1	2	1	2
8		2	2	2	1	2	1	*	1	2	2	
9		2	2	2		2	1	1	*	1	2	2
10		3	3	3		3	3	3	1	*	1	1
11		3	3	3	3		1	3	3	1	*	1
12					3	3	3		3	1	1	*

X_1

Cooperation Matrix

Cooperation Graph: the label is simplified into edge types (dashed line for 2 and dotted-dashed line for 3



Cooperation Graphs

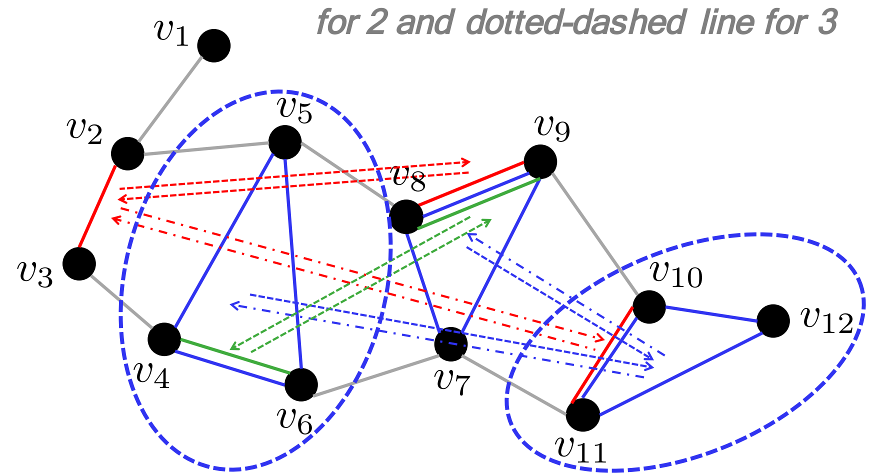
- **The aforementioned matrix is referred to as a *cooperation matrix***
  - ➢ Construct the cooperation graph from the cooperation matrix
    - Each cycle represents a pair of edges/triangles, with an arrow pointing from one to the other, labelled with the cooperation level

Y_2

	1	2	3	4	5	6	7	8	9	10	11	12
1	*	1										
2	1	*	1		1			2	2	3	3	
3		1	*	1				2	2	3	3	
4			1	*	1	1		2	2	2	2	
X_2 5		1		1	*	1		1			2	2
6				1	1	*	1	2	2	2		2
7						1	*	1	1	2	1	2
8		2	2	2	1	2	1	*	1	2	2	
9		2	2	2		2	1	1	*	1	2	2
10		3	3	3		3	3	3	1	*	1	1
11		3	3	3	3		1	3	3	1	*	1
12					3	3	3		3	1	1	*

Cooperation Matrix

Cooperation Graph: the label is simplified into edge types (dashed line for 2 and dotted-dashed line for 3



Cooperation Graphs

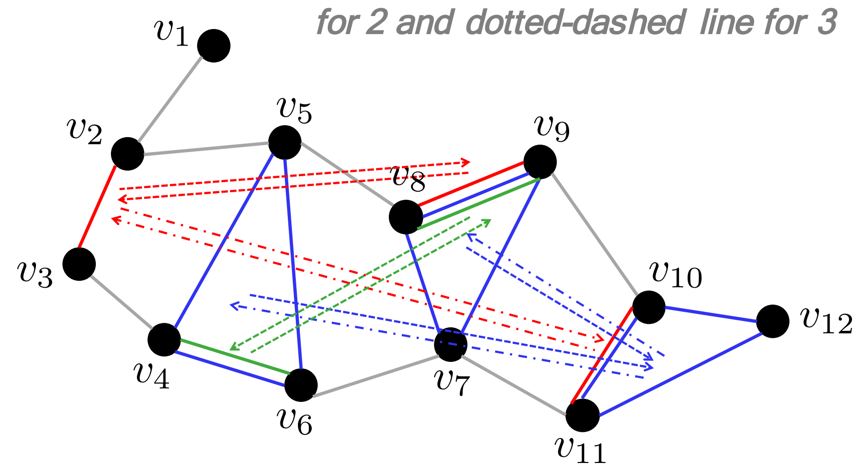
- **The aforementioned matrix is referred to as a *cooperation matrix***
  - ➢ Construct the cooperation graph from the cooperation matrix
    - Each cycle represents a pair of edges/triangles, with an arrow pointing from one to the other, labelled with the cooperation level

Cooperation graph depicts the cooperation of information, i.e., how the information is coupled in the coded DSN

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	*	1										
2	1	*	1		1			2	2	3	3	
3		1	*	1				2	2	3	3	
4			1	*	1	1		2	2	2	2	
5		1		1	*	1		1			2	2
6				1	1	*	1	2	2	2		2
7						1	*	1	1	2	1	2
8		2	2	2	1	2	1	*	1	2	2	
9		2	2	2		2	1	1	*	1	2	2
10		3	3	3		3	3	3	1	*	1	1
11		3	3	3	3		1	3	3	1	*	1
12					3	3	3		3	1	1	*

Cooperation matrix

Cooperation graph:
the label is simplified into edge types (dashed line) for 2 and dotted-dashed line for 3



Constructions Over Compatible Graphs

- **Compatible graphs**
  - Cooperation graphs that adopts the following construction
    - Higher level cooperation: $\mathbf{B}_{i,j}\mathbf{V}_{j;l}$ $(k_i \times \eta_{j;l}, \eta_{j;l} \times r_j)$
    - $\mathbf{E}_{i;l;t}$ $(k_i \times \gamma_{i;t})$, $\mathbf{B}_{i,j} = \left[\mathbf{E}_{i;l;t}, \mathbf{0}_{k_i \times (\eta_{j;l} - \gamma_{i;t})}\right]$, $t \in T_{i;l}, j \in Y_{t;i}$
    - $T_{i;l}$: indices of all cycles that provide extra parities in the l-th level cooperation of node v_i

$$\mathbf{T}_i = \left[\begin{array}{c|c|c|c|c} \mathbf{A}_{i,i} & \mathbf{B}_i & \mathbf{E}_{i;2} & \ldots & \mathbf{E}_{i;L_i} \\ \hline \mathbf{U}_i & \multicolumn{4}{c}{} \\ \cline{1-1} \mathbf{V}_{i;2} & \multicolumn{4}{c}{} \\ \cline{1-1} \vdots & \multicolumn{4}{c}{\mathbf{Z}_i} \\ \cline{1-1} \mathbf{V}_{i;L_i} & \multicolumn{4}{c}{} \end{array}\right]$$

$$\mathbf{E}_{i;l} = \left[\begin{array}{c|c|c} \mathbf{E}_{i;l;t_1} & \ldots & \mathbf{E}_{i;l;t_{|T_{i;l}|}} \end{array}\right]$$

$$\mathbf{B}_i = \left[\begin{array}{c|c|c} \mathbf{B}_{i,j_1} & \ldots & \mathbf{B}_{i,j_{|\mathcal{M}_i|}} \end{array}\right]$$

  - ECC Hierarchy

$$\mathbf{m}_i \xrightarrow{\mathbf{B}_{i,j}} \underbrace{\mathbf{m}_i\mathbf{B}_{i,j}}_{\text{Cross parities}} \xrightarrow{\mathbf{U}_j(\mathbf{V}_{j;l})} \underbrace{\mathbf{m}_i\mathbf{B}_{i,j}\mathbf{U}_j(\mathbf{m}_i\mathbf{B}_{i,j}\mathbf{V}_{j;l})}_{\text{Encoded cross parities}}$$

$$d_{i,0} = r_i - \delta_i - \textstyle\sum_{l=2}^{L_i} \eta_{i;l},\ d_{i,1} = r_i + \sum_{v_j \in \mathcal{M}_i} \delta_j,$$
$$d_{i,2} = r_i + \textstyle\sum_{v_j \in \mathcal{M}_i} \delta_j + \sum_{2 \le l' \le l, t \in T_{i;l'}} \gamma_{i;t}$$

Constructions Over Compatible Graphs

- **Objective**: Cross parities at each level can be removed by accessing the neighboring nodes only

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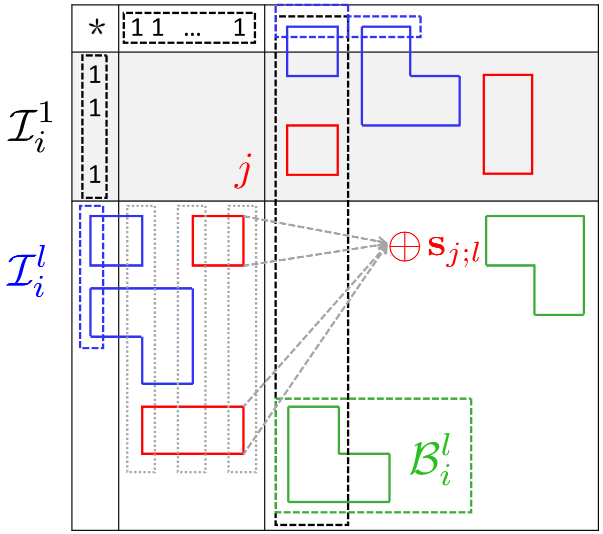

Constructions Over Compatible Graphs

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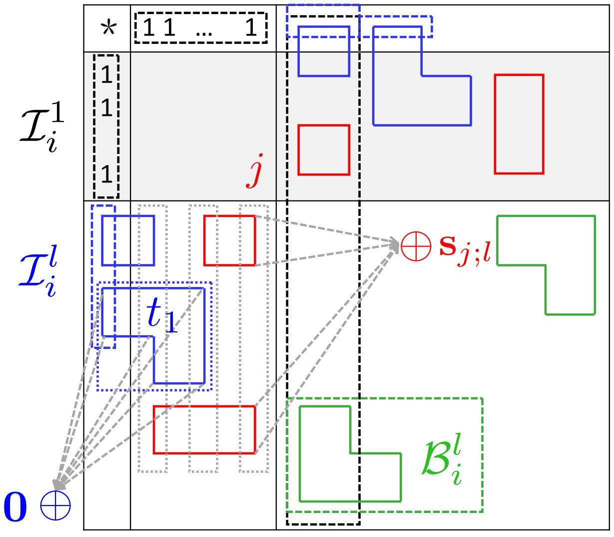
  - The summation of cross parities generated from all nodes at each cycle is a zero vector on $\mathrm{GF}(2^n)$
  - Sum ($\mathbf{s}_{j;l}$) of l-th level cross parities at node v_j in are obtained if v_j is recovered



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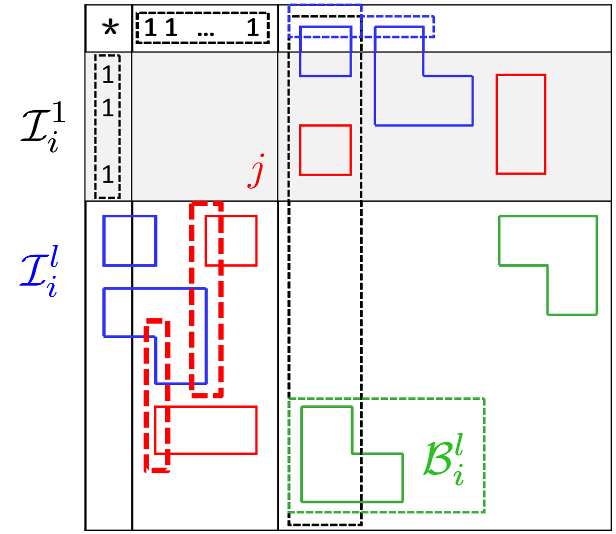

The cycle group containing $\mathcal{J}_i^l$ *such that its intersection with column j is the union of some* $\mathcal{J}_j^{l'}$ is completely contained in columns spanned by $\mathcal{J}_i^l$

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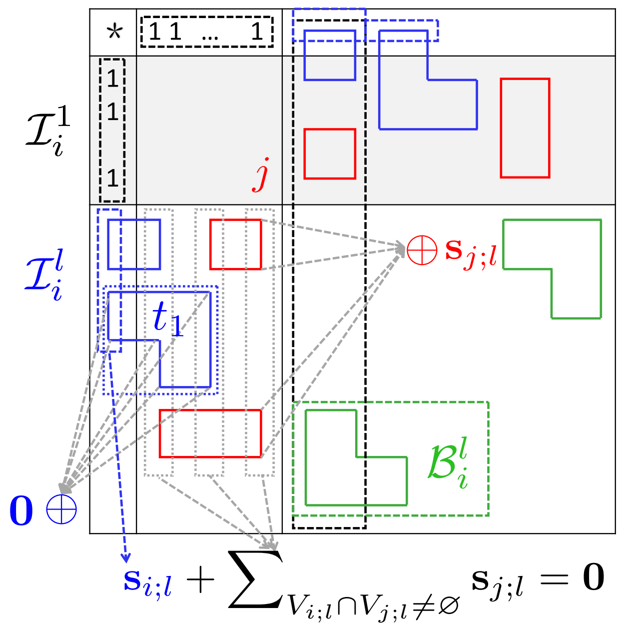
    - $\mathbf{s}_{i;l}$ is the summation of all $\mathbf{s}_{j;l}$'s



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Outline

- **Introduction**
  - Motivation and Model
  - Existing work
- **Cooperative Data Protection**
  - ECC hierarchy
  - Single-level cooperation
- **Multi-level cooperation**
  - Cooperation graphs and compatible graphs
  - Construction over compatible graphs
- **Conclusion**

Conclusion

- **Main contributions**
  - A hierarchical coding framework that provides topology-aware cooperative data protection for DSNs
  - The framework supports scalability and flexibility
  - Our scheme achieves faster recovery speed and corrects more flexible erasure patterns

- **Follow-up work (already done)**
  - Analysis of recoverable erasure patterns
  - Algorithms that search for cooperation graphs in a network

- **Future work**
  - Codes that support non-locally decodable neighboring nodes
  - Error correction for latency-sensitive devices at the edge

Q&A

Thank you!

Q&A

* For further reference

[1] S. Yang et al., "*Topology-aware cooperative data protection in blockchain-based decentralized storage networks*", in Proc. IEEE ISIT, Los Angeles, CA, USA, Jun. 2020, pp. 622-627

[2] S. Yang et al., "*Hierarchical coding for cloud storage: topology-adaptivity, scalability, and flexibility*", submitted to IEEE TIT, 2020, available on https://arxiv.org/abs/2009.09146